Size-dependent policies, talent misallocation, and the return to skill*

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Abstract

We study the allocation of talent in knowledge-based hierarchies subject to a payroll tax that increases with firm size. The policy distorts occupational choices, creating talent mismatch and smaller firms. We calibrate the undistorted model to the U.S. economy, and conduct two counterfactual exercises. A 2.3% tax on firms with 50 or more employees generates output losses of 7%. Returns to skill decrease 27% (5%) for workers (managers). A continuously-increasing tax, calibrated to match Mexico's high self-employment, implies output losses of 12%, returns to skill that are 60% (7%) lower for workers (employers), and an average tax of 61%.

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1 Introduction

Workers in large firms are, on average, more skilled than workers in smaller firms. Further, larger firms tend to be run by more educated entrepreneurs, relative to smaller firms. If more talented individuals form larger firms, which, in turn, tend to have a larger share of highly skilled workers, then policies that affect firms of different sizes differently will distort the allocation of talent, as well as the return to skill. In this paper we argue, in particular, that firm regulations which favor small businesses misallocate talent throughout the entire economy, and lower the return to skill.

The relationship between skill composition and firm size holds across countries, and for different measures of skill and size of the firm. Figure 1 shows that, in the U.S., the larger the size of the firm, the larger the fraction of relatively educated employees, and the smaller the fraction of employees with fewer years of schooling.[1] Figure 2 shows that more formally educated business owners tend to hire more employees, although this relation is more noisy than in the case of employees. Figure 3 shows that in Mexico large firms exhibit, on average, a higher fraction of more formally educated employees. In Latin America, average years of formal education for workers in a micro firm (1-5 workers) are 1.2 years below the national average. Further, schooling for the average worker in a medium-size firm (6-50 workers) is 0.3 years higher than the national average, while average schooling for workers in large firms (above 50 workers) is 0.84 years higher than the national average. Larger firms in Latin America also tend to be managed by more educated business owners, as seen in Figure 4.[2]

[1]Unless we specify otherwise, whenever we use micro-data from labor market surveys, we restrict our sample to full-time, full-year male workers in metropolitan areas, ages 25 through 64, not enrolled in school.

[2]Other authors also report evidence on positive sorting in size and skill for both workers and managers. Headd (2000) reports that in the U.S. small firms are more likely to employ workers with a high school diploma or less, whereas workers with at least some college are more likely to work in larger firms. Also for the U.S., Cardiff-Hicks et al. (2014) find that higher quality workers are sorted into large firms and large establishments in retailing. Fox (2009) documents evidence for Sweden consistent with hierarchical matching, while Busso et al. (2012) document a positive relation between cognitive skills and firm size for both employers and employees in OECD and Latin American countries.

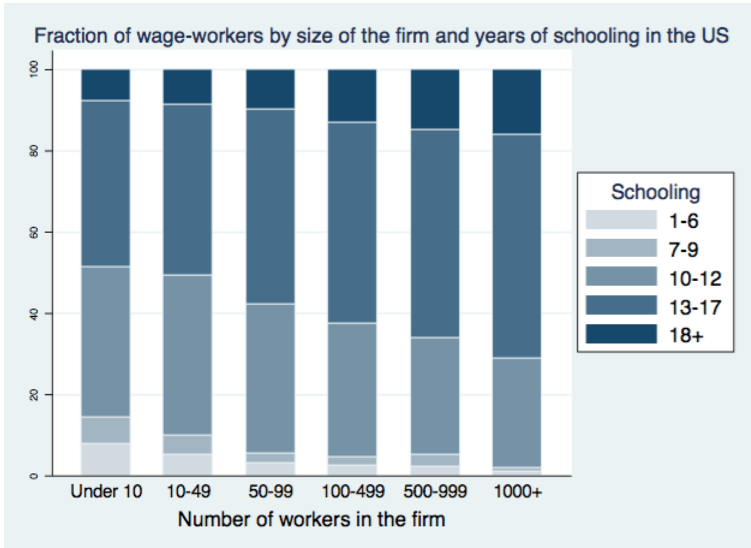

Figure 1

Skill Composition of the Firm in the U.S.

Source: Author's calculations using the 2014 March Supplement of the Current Population Survey (CPS).

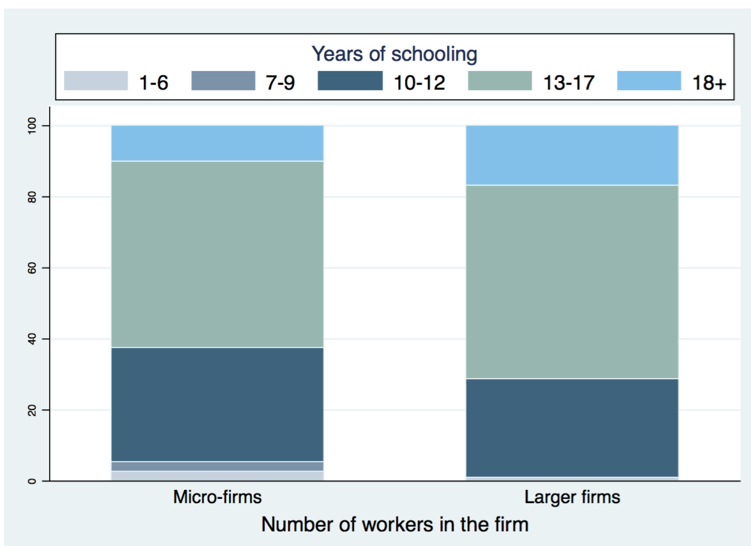

Figure 2

Distribution of Employers by Years of Schooling and Size of the Firm in the U.S.

Source: Author's calculations using the 2014 March Supplement of the CPS.

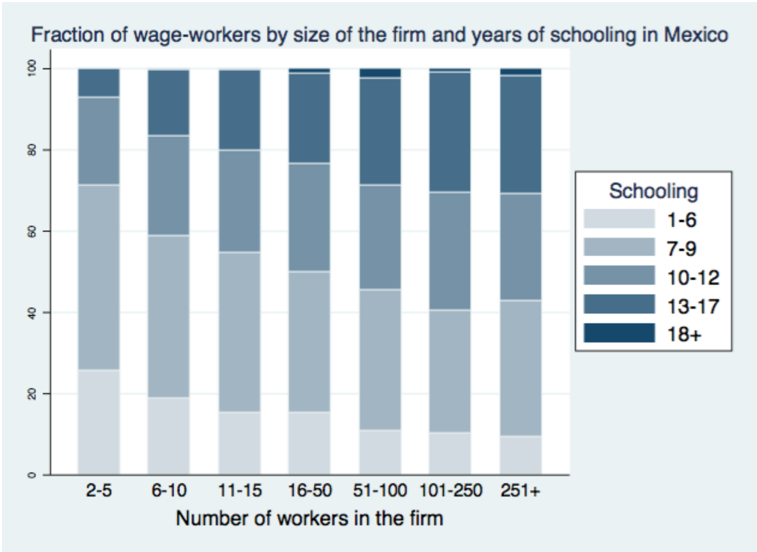

Figure 3
Skill Composition of the Firm in Mexico
Source: Author's calculations using the National Survey of Occupations and Employment (ENOE, by its Spanish acronym), 2014-Q3.

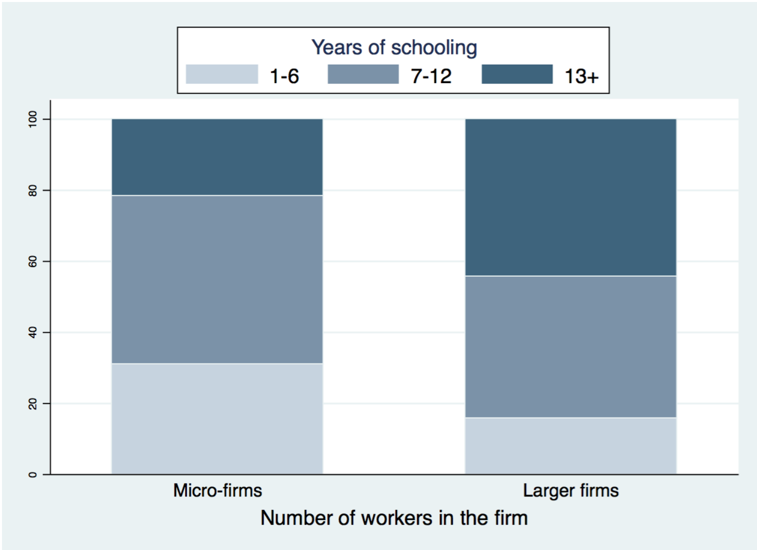

Figure 4
Distribution of Employers by Years of Schooling and Size of the Firm in Latin America
Source: Author's calculations using the Inter-American Development Bank Harmonized Household Surveys.

In this paper, we study the allocation of talent in a knowledge economy when firms are subject to size-dependent regulations, which we model as a tax on labor that increases with firm size. To this end, we embed the hierarchical model of Garicano and Rossi-Hansberg (2004, 2006) into a production economy subject to decreasing returns to scale, as in Lucas (1978). This extension allows us to analyze how individuals with heterogeneous abilities are sorted into occupations and into firms of varying sizes, while delivering a realistic distribution of firm sizes. Under a specific parameterization detailed below, our undistorted environment generates closed-form solutions for all equilibrium objects, including a Pareto firm-size distribution which we can calibrate to closely match that of the U.S. economy.

In the model, the skill of an agent completely determines his occupation, the quality of his match, as well as the size of his firm, and the reward for marginally increasing his type—the return to skill—derives from matching with more talented workers in larger firms. The size-dependent tax encourages employers to constrain the size of their firm, and discourages their wage workers from fully exploiting their talent. The policy also reallocates the marginal individual in each occupation, which in turn changes the sorting of the infra-marginal individuals. As a result, employers coordinate less-talented employees, and run smaller firms, compared to the equilibrium without distortions.

In our model, both talent misallocation and a lower return to skill originate from the same source: distortion on occupational choices, which reorganizes production by re-sorting everyone within occupations, and creates talent mismatch. To understand the magnitude of these effects, we calibrate an undistorted version of our knowledge-based economy to match some features of the U.S. economy, and then conduct two counterfactual exercises. First, we introduce a perfectly-enforced payroll tax of 2.3% on firms with 50 or more employees—as the one estimated by Garicano et al. (2016) using French firm-level data and a model with homogeneous workers. We find that this tax generates output and productivity losses of 7%. The returns to skill drop by 27% for workers, and 5% for managers. When we shut down worker heterogeneity and positive sorting in quality—which would correspond to a model similar to Garicano et al. (2016)—output and productivity losses are just 1%.[3]

In the second counterfactual experiment, we calibrate a continuous, smooth tax function that increases with firm size, so as to match the high share of self-

[3] In their estimates for France, Garicano et al. (2016) find output losses that range between 2.2% and 2.7%.

employment observed in Mexico (around 21%).[4] The output and productivity losses implied by the observed allocation of talent in Mexico are 12%. The returns to skill drop by 60% for workers, and 7% for managers. The average tax rate under this policy is 61%, which we take as a measure of the efficiency wedge implied by the observed allocation of talent in Mexico, vis-à-vis the U.S.

We make two contributions. First, we show that the skill composition of the firm, which has remained absent from the literature on misallocation and TFP, is crucial to fully understand the effects of firm-specific distortions. In doing so, we contribute to the vast literature spurred by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Further, by fully considering the role of the skill distribution, our work also contributes to the literature on size-dependent policies. For instance, the works of Braguinsky et al. (2011), Garicano et al. (2016), and Guner et al. (2008), consider heterogeneity in managerial skill, but this talent becomes useless if agents choose to work for a wage. The evidence on positive sorting in quality and quantity discussed above suggests potentially larger aggregate effects through talent misallocation. In a closely related paper, Alder (2016), shows that deviations from positive sorting between CEO and projects (firms) can have sizable aggregate effects, depending on the degree of complementarity between projects and managers, as well as the correlation between mismatch and project quality. Our paper differs from Alder's in that we focus on the effects of size-dependent distortions on talent misallocation across the entire skill distribution—including wage workers, the self-employed, and managers—as opposed to the allocation between heterogeneous managers and projects of varying quality.

Our second contribution is showing that size-dependent regulations could significantly lower the average return to skill in the economy. Our model features superstar effects, in the sense that the earnings schedule is convex in ability, as in Rosen (1981) and, more recently, Scheuer and Werning (2017). Therefore, because the best managers are matched with the best wage workers, size-dependent distortions act as an increasing marginal tax schedule that disproportionately lowers the return to skill for the most able workers and managers.

Even though our model does not feature endogenous skill formation, as in Bobba et al. (2017), it still has important implications for the role of human capital and

[4] While Mexico does not have a specific size-dependent statutory policy, its large informal sector populated by small firms, as well as the absence of "bunching" in the size-distribution of firms, as documented by Hsieh and Olken (2014), suggest the existence of implicit size-dependent distortions that are more continuous in nature.

industrial policies in the process of development. Our model predicts misallocation of talent and a lower return to skill as consequences of size-dependent policies for a given distribution of skill. That means that even if individuals became more skilled for other reasons, they would still be misallocated as long as the size-dependent policies persisted. This prediction is in line with the evidence for Mexico presented in Levy and López-Calva (2016), who document a growing mismatch between the supply and demand for skilled labor: while the amount of available skilled workers has grown, the earnings of more educated workers have decreased. While Levy and López-Calva (2016) suggest a link between size-dependent policies and the return to skill, our paper is the first to formalize such link. Our results are also consistent with the long-lasting effects of development policies highlighted by Buera et al. (2013), and with the negative effects of costly formalization on the demand for skilled labor presented in D'Erasmo et al. (2014).

Our analysis does not model the firm's incentives to evade the law as in Rauch (1991), Leal (2014), or López (2017), nor does it solve for the optimal governmental response. Instead, we assume that the government collects taxes mainly from large firms but not small firms, and study the effects of this policy in the labor market, specifically on the allocation of workers and equilibrium prices.

2 The allocation of talent: Knowledge hierarchies and decreasing returns

The basic environment is an extension of the frictionless, general equilibrium, continuous assignment model of Garicano and Rossi-Hansberg (2004, 2006), which we embed into an economy subject to decreasing returns to scale, as in Lucas (1978). Unlike Sattinger (1993), here the assignment is many-to-one, the densities in each sector are endogenous, and there is the option of not matching.

Individuals differ in a single trait—call it talent—and own one unit of time. They choose the use of their time and talent that maximizes their earnings. They can either produce alone or in a team (firm or business) with other workers. They work together to specialize either in managerial activities—running the business—or in production activities—working as one of the firm's employees—and thus exploit complementarities in production. Individuals have then three options: to produce alone, to run a business hiring others, or to work for a wage for someone else. The formation of firms in the model is also endogenous—the entrepreneurs optimally select the size of their firm, as well as the quality of their employees, whereas the wage workers optimally select which of the continuum of teams to join.

To produce in this economy, workers solve problems which vary in their difficulty, z, according to some density $g(z)$. Skill is cumulative—a worker of skill z can solve all problems of difficulty less than or equal to z. Workers draw and attempt to solve one problem in their unit of time and produce only if they know the answer. The (expected) earnings of worker z are therefore the percentage of problems he is able to solve: $G(z)$. The skill endowment z varies continuously in the population according to the (given) skill distribution $F(z)$, with support $[L, H]$, and density $f(z)$.

Agents can also form teams, each consisting of identical production workers and one manager. In these teams, the manager attempts to solve the problem whenever his production workers do not know the answer, and production workers do not interact with each other. More precisely, a team with n employees draws n problems, and the (expected) output of the team is the percentage of tasks that a manager with skill z_m is able to solve in his n units of time,

$$y = G(z_m)\, n^{\alpha},$$

where $\alpha \in (0, 1)$ denotes the degree of decreasing returns to scale in the use of time, as in Lucas (1978).

Communication in teams is costly: employees of skill z_p will ask the manager with probability $1 - G(z_p)$, and solving these problems costs a fraction $h(z_p)$ of the manager's time, per worker. We assume that these costs are bounded: $h(L) = \overline{h}$ and $h(H) = \underline{h}$, with $0 < \underline{h} < \overline{h}$. Thus, communication in teams is always costly: communicating with a worker who knows all the answers still takes a fraction of the manager's time. Communication costs then further limit the entrepreneur's span of control, or firm size, n. Within his unit of time, a manager is able to coordinate at most ${}^{1}/_{h(z_p)}$ workers, and this way

$$nh(z_p) \leq 1.$$

In other words, if wage workers are of quality z_p, a manager can coordinate at most $n(z_p)$ employees. The problem of a manager with ability z_m is to choose the quality of his employees, z_p, and the size of his firm n, so as to solve

$$R(z_m) = \max_{z_p,n} \; G(z_m)\, n^{\alpha} - w(z_p)\, n,$$
$$s.t. \;\; n \leq n(z_p), \tag{1}$$

where $w(z_p)$ denotes the equilibrium wage rate, and

$$n(z_p) \;\; = \;\; \frac{1}{h(z_p)}.$$

Without any distortion, the inequality constraint will always bind: from the manager's point of view, firm size l is feasible hiring wage workers of any type above $n^{-1}(l)$, but costs are minimal when employing workers of skill $n^{-1}(l)$, who earn the lowest wage, and that is true for all firm sizes. Firm sizes in the model are therefore simply a function of the knowledge in the bottom layer of the firm. Thus, we can rewrite output as

$$y(z_p, z_m) = G(z_m)\,[n(z_p)]^{\alpha}, \tag{2}$$

and managerial rents are thus

$$R(z_m) = \max_{x} G(z_m)\,[n(x)]^{\alpha} - w(x)\, n(x). \tag{3}$$

Individuals take wages and managerial rents as given, and choose the occupation that yields the highest earnings given their skill:

$$\max\{w(z), G(z), R(z)\}. \tag{4}$$

The equilibrium consists of an assignment of individuals into occupations and into firms, as well as wages and managerial rents, such that no agent desires to switch to another occupation or firm.[5]

The equilibrium exhibits positive sorting in both quality and quantity, as in Garicano and Rossi-Hansberg (2004, 2006) and Garicano and Hubbard (2012). Specifically, the best managers match with the best wage workers to form the largest teams, the second-best managers match with the second-best wage workers to form the second-largest firms, and so on. The smallest firms in the market are thus the match of the least-skilled entrepreneurs to the least-skilled wage workers. The equilibrium also exhibits perfect stratification of individuals into occupations based on their skill: the less skilled agents become production workers, those in the middle produce alone, while the most skilled ones work managing others. Figure 5 shows the equilibrium allocation of workers into occupations according to their skill level.[6]

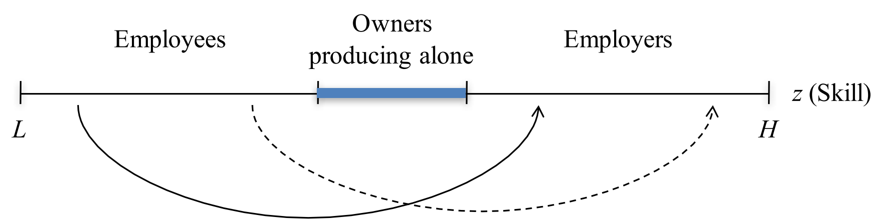

Figure 5
Allocation of Workers across Occupations

To find the equilibrium, we follow the algorithm in Sattinger (1993), as described

[5]In the setup in Garicano and Rossi-Hansberg (2004), $\alpha = 1$ and $h(z_p) = b(1 - G(z_p))$. In that case,

$$\lim_{z_p \to H} n(z_p) = \infty$$

and because output exhibits constant returns to scale, managers have incentives to set firms as large as possible by matching with the worker right next to them, which in turn can only occur by adding additional layers, and the model has no equilibrium. High communication costs—the value of b—is what prevents them from adding more layers, but high values of b significantly limit the support of the equilibrium distribution of firm sizes. In our alternative setup the incentives to set large firms are diminished by $\alpha < 1$ and, in addition to bounding $n(z_p)$, there is a broader range of values of b which allow for an equilibrium to exist. Moreover, our setup allows us to readily place the effects of the size-dependent tax under hierarchical matching within the current literature in development economics on firm-level distortions.

[6]Unlike the equilibrium in the original framework, the equilibrium in this model could exhibit segregation of workers, as in Kremer and Maskin (1996), in which the top agents match together whereas workers at the bottom produce alone. Whether the equilibrium entails segregation or not depends on the sensitivity of earnings to skill in the outside option, the skill distribution, and the strength of complementarities in production. In our analysis we focus on the equilibrium without segregation as the one displayed in Figure 5.

by Garicano and Rossi-Hansberg (2004, 2006). We conjecture the existence of two thresholds, z_1 and z_2, such that agents with skill $z \leq z_1$ optimally become wage workers, agents with skill $z_1 < z < z_2$ optimally choose to produce alone, and agents with skill $z \geq z_2$ optimally choose to manage others. The first step is to find the equilibrium assignment function $z_m = m(z_p)$, which denotes the manager z_m that corresponds to workers of skill z_p.

In equilibrium the market for managers clears: for all $s \in [L, z_1]$ it has to be the case that

$$\int_L^s \frac{f(x)}{n(x)} dx = F(z_m(s)) - F(z_2), \tag{5}$$

which implies

$$\frac{f(s)}{n(s)} = f(m(s)) \frac{dm}{ds}. \tag{6}$$

We solve for $m(z_p)$ using equation 6, with $m(L) = z_2$—which states that the worst wage workers are matched with the worst employers—as the boundary condition. The second step is to combine the first order condition (FOC) of the entrepreneur's problem with the equilibrium assignment function to find the equilibrium wage function. The firm's problem is to select z_p to maximize

$$R(z_m) = G(z_m)[n(z_p)]^{\alpha} - w(z_p) n(z_p).$$

The FOC is

$$G(z_m) \alpha [n(z_p)]^{\alpha-1} \frac{dn}{dz_p} = \frac{dw}{dz_p} n(z_p) + w(z_p) \frac{dn}{dz_p}. \tag{7}$$

We then plug in $z_m = m(z_p)$ from step one above, and solve the differential equation for $w(z_p)$ using the equilibrium condition $w(z_1) = G(z_1)$, which states that the best wage workers are indifferent between working for a wage or producing alone.

The final step is to pin down the constants of integration in the two previous steps to completely characterize the assignment function $z_m = m(z_p)$, and the earnings functions $w(z)$ and $R(z)$. To do so, we solve $m(z_1) = H$ and $R(z_2) = G(z_2)$. The former condition states that the best managers match with the best workers, while the latter states that the worst managers are indifferent between running a team and producing alone.

In what follows, we rely on numerical examples to illustrate the properties of the model when we introduce size-dependent distortions. We then use a parametric example that delivers closed-form solutions for the undistorted case to examine the effects of specific policies.[7]

3 Effects of size-dependent regulations

3.1 Perfect enforcement of a size-dependent payroll tax

In this section we study the general equilibrium effects of the following policy: firms which hire more than N workers pay a tax $\tau \in (0, 1)$ on their wage bill. Garicano et al. (2016) develop and estimate a simpler version of this problem, in which wage workers are homogeneous in their abilities. This step-tax policy is a limiting case of an imperfectly enforced tax, which we study in the next section.

In our numerical exercises we find that the size dependent tax results in the following: i) a lower average firm size, ii) worse average wage worker quality, iii) lower earnings and returns to skill for the best wage workers and the best managers, iv) more self-employment, driven mainly by the best workers who choose to produce alone after the tax, and v) less wage employment.

The size-dependent tax reallocates individuals in the middle of the skill distribution into another occupation. It does not distort the occupational choices of the least and the most talented individuals, but lowers the average quality of those who work for a wage. Moreover, it encourages employers within a range above the size threshold in the managerial skill distribution to constrain the size of their firm to avoid the tax, and discourages their wage workers from fully exploiting their talent. In other words, there is a set of managers and workers that are constrained by the size-dependent tax.

The tax breaks the positive assortative matching of wage workers and managers in the constrained firms. Wage workers in this segment of the market are identical from the point of view of the managers, and employees are indifferent to the skill of their manager because in equilibrium they earn the same wage, regardless of the firm

[7] The positive sorting of workers is guaranteed as long as the second-order condition holds. We do not prove the uniqueness of the equilibrium—which Garicano and Rossi-Hansberg (2006) do for the particular functional forms they use— but we fail to find equilibria with different stratifications in our numerical exercises.

they join. Thus, for those constrained by the size-dependent tax, positive sorting among workers and managers is only one of infinitely many possible outcomes. The tax does not break the positive sorting of workers in unconstrained firms, but it misallocates managerial talent in this segment of the market—the tax distorts the selection into occupations, which alters the rank of each worker within each occupation. This, in turn, distorts the equilibrium assignment of managers to wage workers. As a result, employers coordinate less-talented employees, and run smaller firms, than in the equilibrium without distortions. This misallocation of talent across the entire skill distribution results in a lower level of output.

The equilibrium with distortions is characterized by six skill thresholds, z_1 through z_6. Define $z_1 \equiv n^{-1}(N)$—the employees' skill that correspond to team size N—using the technology which groups workers into teams. Consider manager z_m. His optimization problem with the size-dependent tax is:

$$\max_{z_p} G(z_m)[n(z_p)]^{\alpha} - \begin{cases} w(z_p)\,n(z_p) & \text{if } n(z_p) \leq N \text{ or } z_p \leq z_1 \\ w(z_p)\,n(z_p)(1+\tau) & \text{if } n(z_p) > N \text{ or } z_p > z_1 \end{cases} \tag{8}$$

The optimality condition varies with the skill of the manager: there is an interior solution for managers of low and high skill, and a corner solution at z_1 for managers in the middle. We use z_5 to stand for the last (most skilled) manager who hires N workers and does not pay the tax. Managers of skill $z_m \leq z_5$ hire $n \leq N$ workers by selecting z_p according to the FOC in equation 7—which corresponds to solving the top panel of equation 8. Similarly, we use z_6 to denote the first (least skilled) manager to pay the tax. Managers of skill $z_m \geq z_6$ choose $n \geq N$ by selecting z_p based on the FOC of the bottom panel of equation 8. Managers of skill z_m in $[z_5, z_6]$ optimally choose N as the size of their firm—for them the marginal benefit of choosing N workers exceeds its marginal cost, whereas the marginal cost of setting $n > N$ is above its marginal benefit.[8] Thus, managers in $[z_5, z_6]$ optimally demand wage workers of skill z_1 to set up firms of size N.

Note that the employees' skill affects the manager's profits through (i) the size of the firm, and (ii) the wage rate. If the size of the firm for all managers in $[z_5, z_6]$ is the same at N, they would employ workers in $[z_1, z_2]$ for some $z_2 > z_1$ if all these wage

[8] If $z_5 = z_6$ then we are in the case where all high-skill managers choose N as their firm size, and no firm pays the tax. We analyze this case at the end of this section.

workers earned the same wage rate (workers of skill below z_1 cannot form teams of size N, whereas for workers of skill above z_1, N is always a feasible firm size). This way, managers in $[z_5, z_6]$ would be indifferent to whom they hire, and wage workers in $[z_1, z_2]$ would be indifferent to the quality of their manager because they would earn the same wage regardless of their match. Therefore, in the equilibrium under the size-dependent policy there is a group of wage workers of skill above z_1 that will be allocated to managers $[z_5, z_6]$ at a constant wage rate, with the endogenous cutoff z_2 as the last of these wage workers and $\overline{N} \equiv n(z_2)$.[9] The skill level z_6 corresponds to the manager indifferent between not paying the tax with a firm of size N, and paying the tax running a larger firm with $\overline{N}$ employees.

Thus, in the new equilibrium there are cutoffs z_1 through z_6 in the skill distribution such that both the set of wage workers and the set of managers are split into three segments. Cutoffs z_3 and z_4 correspond to the most skilled worker—who is indifferent between working for a wage and being self-employed—and the least able manager—who is indifferent between being self-employed and running the smallest firm. As illustrated in Figure 6, within each occupation, individuals at the bottom work in firms with less than N employees, those in a segment above the threshold work in firms with exactly N employees, and those at the top work in firms with at least $\overline{N}$ employees, for some endogenous size $\overline{N} > N$. Workers do not set up firms of sizes $(N, \overline{N})$; they instead constrain the size of their firm to N.

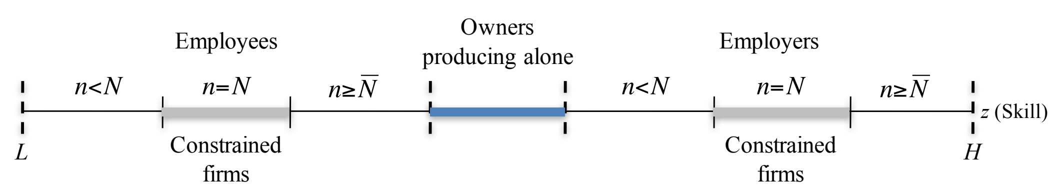

Figure 6
Equilibrium Allocation of Workers across Occupations and Firm Sizes under a Perfectly-enforced Size-dependent Payroll Tax

The assignment functions for each segment of wage workers in the unconstrained firms are obtained equating the supply and demand for managers using the equivalent of equation 6. The assignment of wage worker in $[z_1, z_2]$ to managers in $[z_5, z_6]$ is indeterminate, as we explain below. The only equilibrium condition for this segment

[9]More formally, $z_1 = z_2$ would require allocating a mass zero of wage worker to managers in $[z_5, z_6]$, which cannot be.

of workers is that the demand for managers for the constrained firms must equal its market supply:

$$\frac{F(z_2) - F(z_1)}{N} = F(z_6) - F(z_5) \tag{9}$$

The wage function for the bottom and top segments of wage workers is obtained combining the equilibrium assignment function with the corresponding FOC; the wage function for the middle segment is flat and set equal to $w(z_1)$. The rents for the business owners are defined as production minus labor costs.

3.1.1 Effect on firm sizes

Under the size-dependent tax managers do not set up firms of sizes in the interval $(N, \overline{N})$; they instead constrain the size of their firm to N. Size-dependent regulations therefore lower the average firm size in the economy, not only because a portion of managers constrain their firm sizes to N, but also because the tax lowers the quality of the best employees—which in turn lowers the largest firm sizes in the market. There is a mass zero of firms of size $(N, \overline{N})$ and a mass point at N.

3.1.2 Effect on occupational choices

The policy affects the occupational choices of individuals at the margin. The tax unambiguously induces the best wage workers to produce alone, as shown in Figure 7. Intuitively, managers have now incentives to form smaller teams, which can only be accomplished by lowering the average quality of the wage worker, sending the best into self-employment.

Figure 7
Reallocation of Workers across Occupations

The qualitative effect of the size-dependent tax on the share of employers is not as clear. On the one hand, a smaller mass of wage workers optimally demands a smaller mass of managers, which tends to push the worst managers in the economy to produce alone (the top managers would outbid them in the competition for wage workers). We call this a *quantity effect*. On the other hand, the lower average quality of wage workers would attract those who used to be the most skilled self-employed into employing others—a *quality effect*. If the quality effect dominates, the smallest firms would become even less productive, as they would now work for managers of less talent relative to the undistorted economy.

3.1.3 Effect on equilibrium assignments

The size-dependent tax breaks the positive assortative matching of wage workers and managers in the constrained firms. Wage workers in this segment of the market are identical from the point of view of the managers–employers prefer one type of worker over another if they group in larger teams, but if all workers in the constrained segment of the market group in teams of the same size, then no type is more attractive than the next. Managers in these firms are thus indifferent to whom they hire. Employees are also indifferent to their type of manager because they earn the same wage regardless of their firm. This segment of the assignment function is therefore indeterminate. Managers could potentially mix employees from different types in a single firm, and positive sorting is only one of infinitely many possible outcomes.

The policy does not break the positive sorting of employees and employers in the unconstrained firms. However, it distorts the selection of workers into occupations, which in turn mismatches managers and wage worker in this segment of the market. More precisely, the tax changes the marginal worker in each occupation, which affects in turn the rank of each individual within each occupation. Because the best wage workers become self-employed, managerial talent is unambiguously wasted as the top managers now run smaller firms with less-talented employees.

Figure 8 illustrates these effects in an economy with a uniform distribution for the difficulty of tasks and an exponential skill distribution. In this particular example, the assignment function is linear, but that does not have to be the case in general.

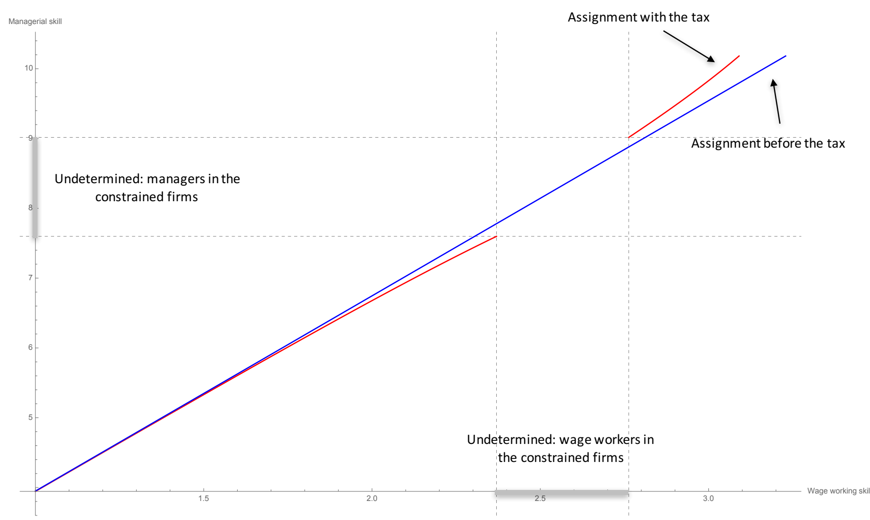

Figure 8

Effect of a Size-dependent Tax on Equilibrium Assignments (τ=0.10 and N=80)

3.1.4 Effect on earnings and the return to skill

The tax unambiguously lowers the return to skill for workers in the constrained firms. Managers in these firms reduce the size of their firm to avoid the tax, but by doing so wage workers lose the ability to differentiate themselves, and therefore lose the reward for marginal increments in their skill (their earnings are now independent of skill), whereas managers lose the reward for pairing with more skilled employees, and their earnings turn linear (as opposed to convex) in managerial skill.

The average return to skill for managers of the unconstrained firms is also lower because they run smaller firms on average, and as a result their earnings profile becomes flatter. Further, the level of earnings of managers at the top is also lower because in addition to running smaller firms, they face a tax. The wage profile becomes flatter on average as well: because managers in this economy now have incentives to form smaller teams, the demand for the least skilled wage workers increases, whereas the demand for the most skilled wage workers decreases. In the new equilibrium, the earnings of the best wage workers are lower, which necessarily results in a flatter wage profile. If wage workers at the very bottom of the skill distribution are matched to managers with more talent relative to their match in the undistorted economy, their earnings could even increase, which would only aggravate the effect on the average return to skill.

Both wage workers and managers in the largest firms then share the burden of the tax (that is, the tax decreases the earnings of those at the very top in both occupations). By exempting small firms from the tax, authorities transfer resources away from those at the top, although not necessarily into wage workers at the bottom.

The effects of the policy on the earnings of the best wage workers and the best employers are displayed in figure 9, which plots (log) earnings before and after a size-dependent tax in an economy with a uniform distribution for the difficulty of tasks, and an exponential skill distribution.

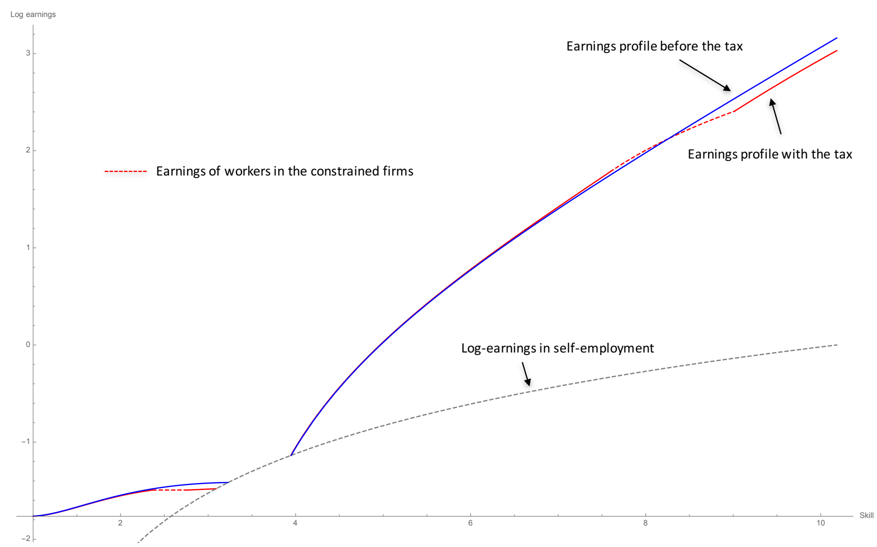

Figure 9
Effect of a Size-dependent Tax on Equilibrium Earnings (τ=0.10 and N=80)

3.1.5 Equilibrium with four thresholds

When complementarities in production are not strong enough (or alternatively, when the tax rate is high enough), all managers above the skill threshold corresponding to firm size N, choose firm size N, and no firm pays the tax. That is, the sets of wage workers and employers splits into only two types: those who set up firms with less than N employees and those who choose exactly N as their firm size. The set of constrained firms corresponds to higher-skill managers and workers. Figure 10 illustrates the equilibrium sorting and firm sizes for the case with four thresholds.

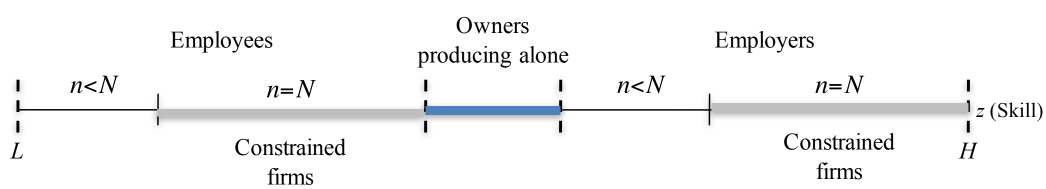

Figure 10
Equilibrium with Four Thresholds

In this equilibrium, wages for the best workers are flat with respect to their skill. Managers in the middle of the skill distribution experience higher rents because they have to pay lower wages, relative to the undistorted equilibrium, whereas the best managers receive lower rents, as they are unable to fully benefit from their high skill by matching with high-skill workers. The individuals who used to be the best wage workers turn to self employment, thus lowering average worker quality. On the other hand, enticed by the lower wages, those who used to be the most skilled self-employed become managers of the smallest firms. Therefore, there is a proliferation of small firms and excessive entry into managerial activities. The lower, flatter wages, and the selection of formerly high-skill wage workers into self-employment are the result of the overall decrease in the demand for skilled labor brought about by the size-dependent tax.

3.2 A general payroll tax

A payroll tax on all firms, regardless of their size, does induce more self-employment, smaller firms, and a mismatch of managers and workers due to the reallocation of workers into self-employment, but earnings do not exhibit flat segments, and despite the (smaller) impact on aggregate output, there is no effect on the return to skill. As in the equilibrium without distortions, there would be two cutoffs such that those at the bottom work for a wage, those in the middle produce alone, and workers at the top produce hiring others. The size constraint would be binding for all firms, and the system of differential equations would be solved as in the case of no distortions discussed above.

3.3 Imperfect enforcement of a general payroll tax

In this section we examine the effects of a general payroll tax whose enforcement increases with firm size. More precisely, we consider a tax function $\tau(n)$, with $\tau'(n) > 0$. Under this policy, the firm's optimization problem is now:

$$\max_{z_p, n} G(z_m) n^{\alpha} - w(z_p) [1 + \tau(n)] n, \text{ s.t. } n \leq n(z_p)$$

and the FOC are

$$
\begin{aligned}
G\left(z_m\right)\alpha n^{\alpha-1}-w\left(z_p\right)\left[1+\tau\left(n\right)+\tau'\left(n\right)n\right]-\mu &\geq 0,\\
-w'\left(z_p\right)\left[1+\tau\left(n\right)\right]n+\mu n'\left(z_p\right) &\geq 0,\\
\mu\left[n\left(z_p\right)-n\right] &= 0,\\
\mu &\geq 0.
\end{aligned}
$$

Where μ is the Lagrange multiplier associated with the optimal size constraint. Note that if the size constraint is binding, prices and assignments in equilibrium are solved for as in the undistorted case. If the size constraint is not binding, then $\mu=0$ and the optimal firm size solves:

$$
\frac{G\left(z_m\right)\alpha}{w\left(z_p\right)} = n^{1-\alpha}\left[1+\tau\left(n\right)+\tau'\left(n\right)n\right],
$$

and from the second FOC it must be true that:

$$
\begin{aligned}
-w'\left(z_p\right)\left[1+\tau\left(n\right)\right]n &= 0,\\
\Longleftrightarrow w'\left(z_p\right) &= 0.
\end{aligned}
$$

That is, the solution requires wages to be independent of skill in this segment of the market, which is exactly the same result as above: if firms constrain their size, then the wages paid to their workers must be independent of their skill. Note, however, that now sizes will depend on managerial skill—much in the spirit of a Lucas (1978) span-of-control model—but positive sorting in these firms is not guaranteed. The qualitative impact of the policy on prices and assignments into occupations and firms in equilibrium is identical to a step-tax policy, and the only difference is in the impact on the firm size distribution: now firms constrain their size not to a single mass point, but to an endogenous range of sizes $\left[N,\widetilde{N}\right]$, and the density would exhibit a *bunching* of firms.

Consider, for example, a Sigmoid tax policy such as the following:

$$
\tau\left(n\right) = \frac{\tau}{1+\exp\left[-\kappa\left(n-N\right)\right]},
$$

where τ is the general tax rate, N is the Sigmoid's midpoint, which represents the threshold around which marginal enforcement significantly increases with firm size. The enforcement parameter κ reflects the steepness of the Sigmoid function around this threshold. Figure 11 shows an example with $\tau = 0.3$, $\kappa = 0.5$, and $N = 20$.

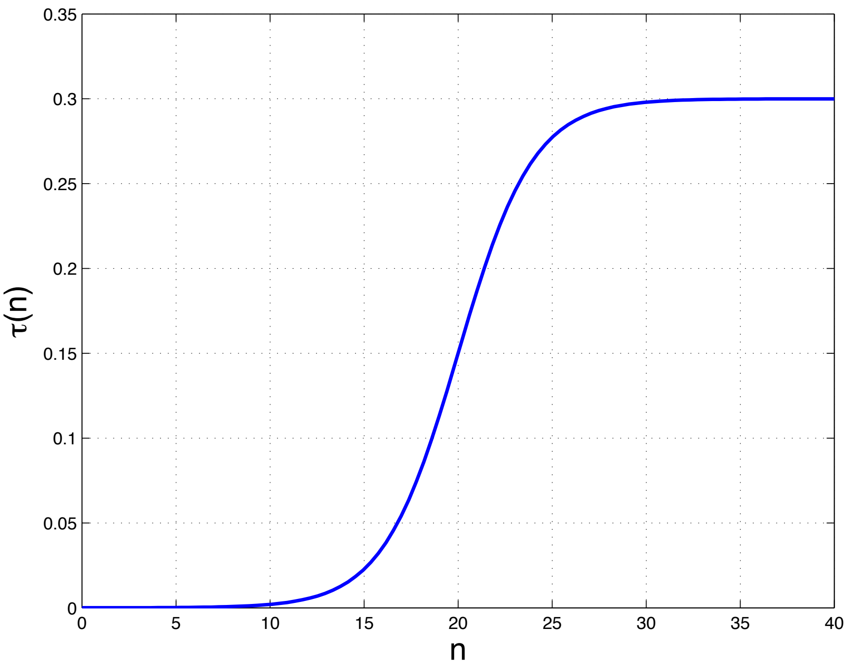

Figure 11
Imperfectly-enforced General Payroll Tax: Sigmoid Tax Function ($\tau = 0.3$, $\kappa = 0.5$, $N = 20$)

Under this policy, the equilibrium firm-size distribution will exhibit bunching in a region of sizes around N. Note that as κ becomes arbitrarily large, this continuous tax approaches the step function discussed above; with values of κ close to 0 the function becomes linear around N, which would mean that firms of all sizes pay some fraction of the tax. In other words, if κ is low enough, which means that the tax function is almost linear around N and the marginal tax burden is not too large around the size threshold, then no firm maximizes at a corner solution (the optimal size constraint is binding for all firms), and the firm size distribution would not display any bunching of firms. In this case there would be no loss in aggregate productivity from the mismatch of managers and wage workers in the constrained firms—only from the reallocation of workers into self-employment—and the qualitative results would be similar to those of a tax on all firms, which we briefly discussed above.

Consider now a tax policy such as the one in López (2017):

$$\tau(n) = \tau[1 - \exp(-\kappa n)]$$

where τ is the general tax rate. As shown in 12, the marginal tax rate under this functional form does not dramatically increase around a particular firm size, and thus the size constraint in the firm's problem always binds. In this case, the equilibrium distribution of firm sizes does not exhibit bunching.

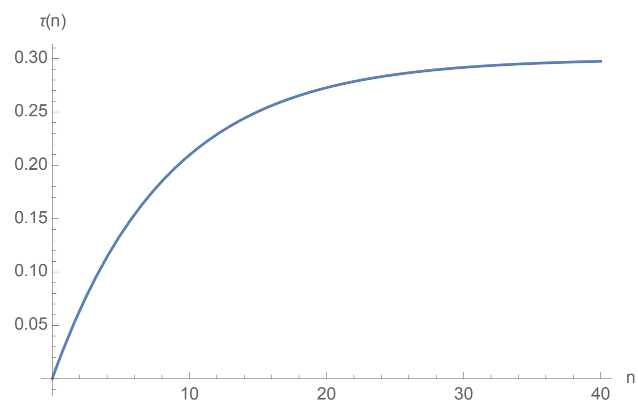

Figure 12
Imperfectly-enforced General Payroll Tax: Smooth Marginal Tax ($\tau = 0.3$, $\kappa = 0.12$)

4 Numerical exercises

4.1 Parameterization and calibration

We now fully characterize a specific parametric example of our benchmark undistorted environment, and calibrate it to match some features of the U.S. economy. We then use the parametric example to conduct two counterfactual experiments, which we detail below.

We assume that the distribution of problems, $G(z)$, is uniform in the interval $[L, H]$. The population skill distribution, $F(z)$, is assumed to be a double-truncated exponential distribution over the interval $[L, H]$, with parameter λ, that is,

$$F(z) = \frac{\exp[-\lambda L] - \exp[-\lambda z]}{\exp[-\lambda L] - \exp[-\lambda H]}$$

Let $q(x) = 1 - G(x)$ denote the probability of asking questions to the manager. Communication costs take the form

$$\begin{aligned} h(x) &= a\exp[bq(x)], \\ &= a\exp\left[b\left(\frac{H-x}{H-L}\right)\right]. \end{aligned}$$

Assumption 1 *Define*

$$\gamma \equiv \frac{a\lambda[H-L]}{\lambda(H-L)+b}.$$

Assume

$$\gamma exp[\lambda(z_2 - L) + b] = 1$$

In this specification and under **Assumption 1**, closed form solutions exist for all the equilibrium objects. All proofs are provided in the Appendix.

Proposition 1 *Under Assumption 1, the equilibrium assignment is a linear function given by*

$$m(z) = z\left[1 + \left(\frac{1}{\lambda}\right)b\left(\frac{1}{H-L}\right)\right] - \left(\frac{1}{\lambda}\right)ln\gamma - \left(\frac{1}{\lambda}\right)b\left(\frac{H}{H-L}\right).$$

The equilibrium wage function is

$$w(z) = \left[\frac{1}{n(z)}\right]\left\{exp[k_3 + k_4 z]\left[\frac{k_2}{k_4} - \frac{k_1}{k_4^2} + \frac{k_1 z}{k_4}\right] + G(z_1)n(z_1) - C(z_1)\right\}.$$

Where $C(z_1) = exp\left[k_3 + k_4 z_1\right]\left[\frac{k_2}{k_4} - \frac{k_1}{k_4^2} + \frac{k_1 z_1}{k_4}\right]$.
The constants k_i, $i \in \{1,2,3,4\}$ *are defined in the Appendix.*

The cutoffs z_1 and z_2 are then

$$
\begin{aligned}
z_1 &= \left[\frac{\frac{1}{a^{\alpha}}k_4 - k_2}{k_1}\right], \\
z_2 &= L + \frac{w\left(L\right)n\left(L\right)}{n\left(L\right)^{\alpha} - 1}\left[H - L\right].
\end{aligned}
$$

The condition needed by Assumption 1 then becomes

$$
\left(-\frac{1}{\lambda}\right)\left[b + \ln\gamma\right] = \frac{w\left(L\right)n\left(L\right)}{n\left(L\right)^{\alpha} - 1}\left[H - L\right].
$$

The specific functional forms in this example also deliver a closed form for the distribution of firm sizes.

Proposition 2 *The firm-size distribution (FSD) is Pareto with power* $\lambda\left[H - L\right]\left(\frac{1}{b}\right) + 1$.

An obvious implication of Proposition 2 is that the power of the Pareto distribution is always strictly greater than one, which stands at odds with the evidence for the U.S. presented in Axtell (2001). The power parameter will be close to one when H is very close to L, and/or when the communication cost parameter b is very high.

We calibrate the model without distortions to match the average firm size, the fraction of wage workers, and the power of the firm size density in the U.S. As in other works similar to ours, such as Garicano et al. (2016), we assume returns to scale equal to 0.8. Table 1 contains all parameter values. Table 2 shows the model's performance.[10]

[10] Data on the distribution of firm sizes for the U.S. come from publicly available statistics from the 2012 Economic Census, published by the U.S. Census Bureau, while data on occupational shares come from our own calculations using the 2014 March Supplement of the CPS.

Table 1
Parameter Values

Parameter	Value
α	.8
a	1×10^{-8}
b	16.5
$s \equiv {}^{L}/_{H}$	.0982
λ	1

Table 2
Calibration Results: Moment Matching

Targeted	Data	Model
Average firm size	17.09	17.09
Share of wage workers	.94	.89
Power of firm size density	1	1.56
Non-targeted	Data	Model
Share of firms of size $<= 19$	.857	.814
Share of firms of size $20 - 49$	.094	.143
Share of firms of size $50 - 99$	.032	.03
Share of firms of size $>= 100$	.026	.013

The model does an overall good job at matching both targeted and non-targeted moments of the firm-size distribution. As pointed out before, the specific functional forms and parametric restrictions required by our closed-form solution to the undistorted environment imply a power coefficient strictly greater than one, which stands at odds with the evidence for the U.S. We find the overall performance of this parametric example to be satisfactory given its analytical convenience.

Figure 13 displays the earnings profiles and occupational choices for this economy.

Indeed, in this example it is the case that $w(z) \geq G(z)$, and $w(z) \geq R(z)$, for all $z \in [L, z_1]$, $G(z) \geq w(z)$ and $G(z) \geq R(z)$ for all $z \in [z_1, z_2]$, and $R(z) \geq G(z)$ and $R(z) \geq G(z)$ for all $z \in [z_2, H]$. Therefore, the conjecture is correct and the allocation is an equilibrium. Figure 14 shows the equilibrium assignment function, which in this benchmark example is linear.

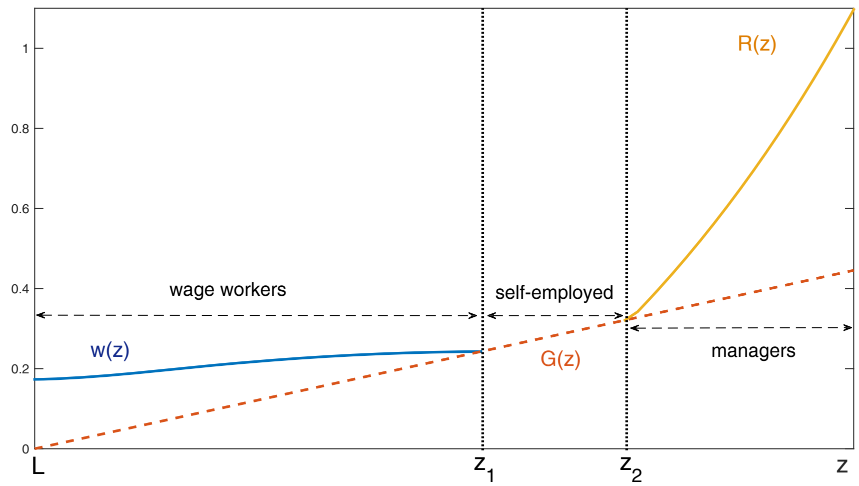

Figure 13

Earnings and Occupational Choices in Benchmark Economy

All parameter values are provided in Table 1.

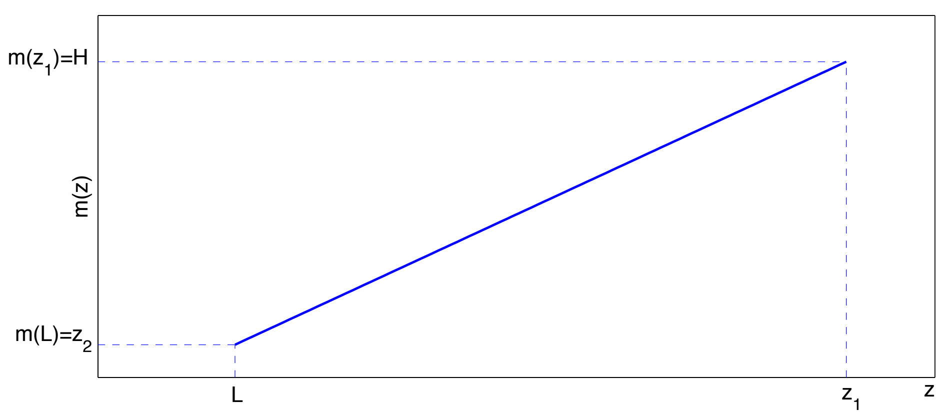

Figure 14

Equilibrium Assignment Function in Benchmark Economy

All parameter values are provided in Table 1.

4.2 Effects of a perfectly enforced size-dependent tax

We introduce a perfectly-enforced payroll tax of 2.3% on all firms with 50 or more employees. The motivation behind this exercise is to compare the output loss predicted by our model to that found in Garicano et al. (2016), who estimate that French regulations on firms with more than 50 employees are equivalent to a 2.3% tax on payroll, using a standard span of control model where workers are homogeneous, and managerial ability is the only source of heterogeneity. Their estimated output loss, using the same span of control parameter value of 0.8, is 2.2% at the lowest and 2.74% at the highest.

Table 3 shows the results of the perfectly-enforced step tax for two cases: our model with hierarchical matching—columns (1) and (2)—and a model with homogeneous workers like the one used in Garicano et al. (2016)—last two columns. Under our model of knowledge hierarchies, self-employment nearly doubles as a result of the tax. All of the increase in self-employment comes from the reallocation of the previously highest-skilled wage workers, who choose to become self-employed after the tax. Figure 15 shows the equilibrium earnings schedule and occupational choices before and after the tax. Demand for high-skill labor drops as a result of the size-dependent tax, disproportionately lowering wages for the most skilled workers. High-skill workers respond to this change by becoming self-employed. There is also a small increase in the share of managers, as a small fraction of previously high-skill self-employed become low-skill firm managers. The output and productivity losses are each 7%. That is, all output losses originate from the efficiency loss brought about by the reorganization of production induced by the size-dependent tax. The returns to skill decrease by 28% for workers, and by 5% for managers.[11] The effects under the assumption of homogeneous wage workers are much smaller. Occupational choices are virtually unchanged by the tax, and output and productivity losses are just 1%.

[11] We calculate the returns to skill by simulating data from the model and running regressions of log of earnings on log of skill for each occupation.

Table 3
Effects of a Perfectly-enforced Size-dependent Tax ($\tau = .023$ and $N = 50$)

	Hierarchical matching		Homogeneous wage workers	
	(1)	(2)	(3)	(4)
	Without tax	With tax	Without tax	With tax
Share of wage workers	.89	.85	.86	.87
Share of self-employed	.06	.09	.11	.11
Share of managers	.05	.05	.03	.02
Output*	1	.93	1	.99
Productivity*	1	.93	1	.99
Return to skill (wage workers)*	1	.72	N/A	N/A
Return to skill (managers)*	1	.95	N/A	N/A

* Relative to corresponding case without tax.

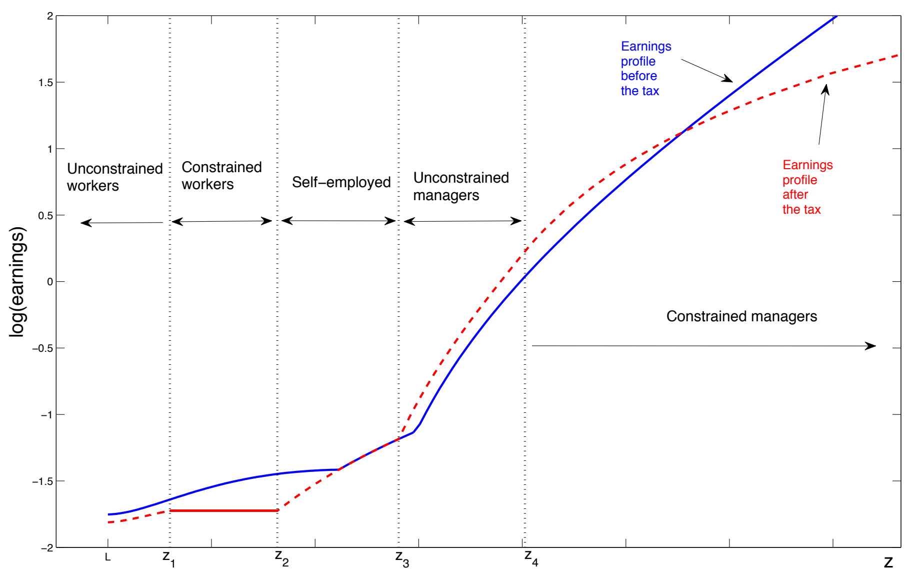

Figure 15
Equilibrium Log Earnings and Occupational Choices before and after the Tax

4.3 Talent misallocation in Mexico

The second counterfactual experiment we conduct asks the question of what would be the size-dependent tax necessary to induce the allocation of talent across occupations observed in Mexico. This exercise is motivated by several factors. According to Pagés (2010), Mexico exhibits the highest TFP losses from misallocation (in the sense of Hsieh and Klenow (2009)) among Latin American countries; even higher than China and India. Further, Mexico has a self-employment share of 21%, one of the highest among OECD countries (OECD, 2017).

Because the distribution of firm sizes in Mexico does not exhibit bunching (Hsieh and Olken, 2014), we introduce a general payroll tax whose enforcement increases smoothly with firm size. More specifically, let τ denote the statutory tax rate, and κ denote the steepness of the enforcement. The tax function we consider is $\tau(1-e^{-\kappa n})$—where n stands for firm size—which is similar to the one in López (2017) discussed above. The strategy is then to calibrate τ and κ so as to match the rates of self-employment (21%) and employers (4.2%) in Mexico.

The calibrated parameter values are $\tau = 0.7325$ and $\kappa = 0.12$, which generate a self-employment share of 21.6% and an employer share of 5.1%. Table 4 shows the results from this experiment. The output and productivity losses are 12%. Again, all the loss in output is the result of productivity losses caused by the shifts in occupational choices. The main mechanism is the same as before: larger firms constrain their size as a result of the tax, which reduces the demand for skilled labor. The corresponding decline in wages for skilled labor induces a shift from wage work to self-employment. In this scenario, the returns to skill decline by 60% for workers, and 7% for managers. The average tax rate is 61%, which we interpret as a measure of the efficiency wedge implied by the observed allocation of talent in Mexico, compared to the U.S.

Table 4
Misallocation of Talent: Counterfactual for Mexico

	(1) Without tax	(2) With tax
Share of wage workers	.89	.73
Share of self-employed	.06	.22
Share of managers	.05	.05
Output*	1	.88
Productivity*	1	.88
Return to skill (wage workers)*	1	.40
Return to skill (managers)*	1	.93
Implied average tax	0	.61

* Relative to corresponding case without tax

5 Concluding remarks

When we consider their effect on the allocation of talent across the entire skill distribution, size-dependent regulations create much larger aggregate losses than those found in similar work where the only source of heterogeneity is managerial talent. Further, we show that size-dependent regulations disproportionately lower the returns to skill for the most able workers and managers. Both the allocation of talent and the returns to skill are prominent concepts in the fields of growth and development—some might say, in the entire discipline of economics—yet remain largely unexplored in the vast literature on firm-specific distortions and misallocation. Our framework allows us to bridge this gap, and its analytical convenience opens many exciting lines of inquiry. Some of these include studying the effects of size-dependent policies on the acquisition of skills, as well as the endogenous evolution of occupational choices and the skill distribution throughout the process of development.

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Appendix

1. Assignment function

The assignment function follows the differential equation

$$\begin{aligned}\frac{f(x)}{n(x)} &= f(m(x))\, m'(x), \\ &= \frac{d}{dx} F(m(x)).\end{aligned}$$

Integrating both sides yields

$$\begin{aligned}\int \frac{f(x)}{n(x)} dx &= F(m(x)), \\ \frac{-a\lambda [H-L]}{(\exp[-\lambda L] - \exp[-\lambda H])(\lambda(H-L)+b)} \left(\frac{\exp[-\lambda x]}{an(x)}\right) + c &= \frac{\exp[-\lambda L] - \exp[-\lambda m(x)]}{\exp[-\lambda L] - \exp[-\lambda H]}.\end{aligned}$$

Where c denotes the constant of integration. Using the boundary condition $m(L) = z_2$ to solve for c yields

$$c = \frac{\exp[-\lambda L] - \exp[-\lambda z_2]}{\exp[-\lambda L] - \exp[-\lambda H]} - \frac{-a\lambda [H-L]}{(\exp[-\lambda L] - \exp[-\lambda H])(\lambda(H-L)+b)} \left(\frac{\exp[-\lambda L]}{an(L)}\right),$$

and therefore

$$m(x) = -\frac{1}{\lambda} \ln \left[1 + \frac{a\lambda [H-L]}{\lambda(H-L)+b} \left(\frac{\exp[\lambda(z_2 - x)]}{an(x)}\right) - \frac{a\lambda [H-L]}{\lambda(H-L)+b} \left(\frac{\exp[\lambda(z_2 - L)]}{an(L)}\right)\right] + z_2.$$

Now define

$$\frac{a\lambda [H-L]}{\lambda(H-L)+b} \equiv \gamma,$$

and rewrite the assignment function using this new constant:

$$m(x) = \left(-\frac{1}{\lambda}\right) \ln \left[1 + \gamma \left(\frac{\exp[\lambda(z_2 - x)]}{an(x)}\right) - \gamma \left(\frac{\exp[\lambda(z_2 - L)]}{an(L)}\right)\right] + z_2.$$

Assume

$$\gamma \exp\left[\lambda\left(z_2 - L\right) + b\right] = 1.$$

If this assumption holds, then we can simplify the assignment function even further to obtain

$$m(x) = x\left[1 + \left(\frac{1}{\lambda}\right) b\left(\frac{1}{H-L}\right)\right] + \left(-\frac{1}{\lambda}\right)\ln\gamma + \left(-\frac{1}{\lambda}\right) b\left(\frac{H}{H-L}\right).$$

2. Wage function

To obtain the wage function we must solve the following differential equation (see FOC in manager's problem):

$$G\left[m(x)\right]\left[\frac{d}{dx}\left[n(x)^{\alpha}\right]\right] = \frac{d}{dx}\left[w(x)\,n(x)\right].$$

Note that

$$G\left(m(x)\right)\left[\frac{d}{dx}\left[n(x)^{\alpha}\right]\right] = \left[k_1 x + k_2\right]\exp\left[k_3 + k_4 x\right],$$

where

$k_1 = \left[1 + \left(\frac{1}{\lambda}\right)\left(\frac{b}{H-L}\right)\right]\alpha\left[\frac{b}{[H-L]^2}\right]\left(\frac{1}{a^{\alpha}}\right)$,

$k_2 = \left[\left(-\frac{1}{\lambda}\right)\ln\gamma + \left(-\frac{1}{\lambda}\right)\left(\frac{b}{H-L}\right)H - L\right]\alpha\left[\frac{b}{[H-L]^2}\right]\left(\frac{1}{a^{\alpha}}\right)$,

$k_3 = -b\alpha\left(\frac{H}{H-L}\right)$,

$k_4 = b\alpha\left(\frac{1}{H-L}\right)$.

Then,

$$\int G\left[m(x)\right]\left[\frac{d}{dx}\left[n(x)^{\alpha}\right]\right]dx = \int \left[k_1 x + k_2\right]\exp\left[k_3 + k_4 x\right]dx$$
$$= \exp\left[k_3 + k_4 x\right]\left[\frac{k_2}{k_4} - \frac{k_1}{k_4^2} + \frac{k_1 x}{k_4}\right],$$

and therefore

$$\exp\left[k_3 + k_4 x\right]\left[\frac{k_2}{k_4} - \frac{k_1}{k_4^2} + \frac{k_1}{k_4}x\right] + c \quad = \quad w(x)\, n(x).$$

Where c denotes the constant of integration. Using the boundary condition $w(z_1) = G(z_1)$ to solve for c yields

$$c \quad = \quad G(z_1)\, n(z_1) - \exp\left[k_3 + k_4 z_1\right]\left[\frac{k_2}{k_4} - \frac{k_1}{k_4^2} + \frac{k_1}{k_4}z_1\right].$$

3. Solve for the equilibrium

We have two final equilibrium conditions:

$$G(z_2)\, n(L)^{\alpha} - w(L)\, n(L) \quad = \quad G(z_2),$$

$$m(z_1) \quad = \quad H.$$

Take the second equation to solve for z_1:

$$\begin{aligned} m(z_1) = H &= a^{\alpha}\left[H - L\right]\left[\frac{z_1 k_1 + k_2}{k_4}\right] + L, \\ H - L &= a^{\alpha}\left[H - L\right]\left[\frac{z_1 k_1 + k_2}{k_4}\right], \\ \left[\frac{\left(\frac{1}{a^{\alpha}}\right) k_4 - k_2}{k_1}\right] &= z_1. \end{aligned}$$

Use the first equation to solve for z_2

$$
\begin{aligned}
G(z_2)\, n(L)^{\alpha} - w(L)\, n(L) &= G(z_2), \\
G(z_2)\left[n(L)^{\alpha} - 1\right] &= w(L)\, n(L), \\
G(z_2) = \frac{z_2 - L}{H - L} &= \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}, \\
z_2 - L &= \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}\left[H - L\right], \\
z_2 &= L + \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}\left[H - L\right].
\end{aligned}
$$

To sum up, then $z_1 = \frac{1}{a^{\alpha}}\left[\frac{k_4 - k_2}{k_1}\right]$ and $z_2 = L + \frac{w(L)n(L)}{n(L)^{\alpha}-1}\left[H - L\right]$. Now recall that from assumption we need:

$$
z_2 = L - \left(\frac{1}{\lambda}\right) b - \left(\frac{1}{\lambda}\right) \ln\gamma.
$$

Then

$$
\begin{aligned}
L - \left(\frac{1}{\lambda}\right) b - \left(\frac{1}{\lambda}\right) \ln\gamma &= L + \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}\left[H - L\right], \\
-\left(\frac{1}{\lambda}\right) b - \left(\frac{1}{\lambda}\right) \ln\gamma &= \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}\left[H - L\right], \\
\left(-\frac{1}{\lambda}\right)\left[b + \ln\gamma\right] &= \frac{w(L)\, n(L)}{n(L)^{\alpha} - 1}\left[H - L\right].
\end{aligned}
$$

4. Firm size distribution

Note that

$$
n(p(m)) = \left(\frac{1}{a}\right) \exp\left[\left(-\frac{1}{\alpha}\right)\left[-k_3 + k_2\left(\frac{k_4}{k_1}\right) - \left(\frac{1}{a^{\alpha}}\right)\left[\frac{m - L}{H - L}\right]\left(\frac{k_4^2}{k_1}\right)\right]\right].
$$

Let $m(n)$ denote the inverse of this function, so

$$
\frac{dm}{dn} = \left[H - L\right] a^{\alpha} \frac{k_1}{k_4^2} \alpha \left[\frac{1}{n}\right].
$$

Recall that the managerial skill distribution follows $\frac{F(m)-F(z_2)}{1-F(z_2)}$. Using the change of variable technique yields

$$\begin{aligned}
\Pr\left(n \leq \overline{n}\right) &= \Pr\left(n\left(m\right) \leq \overline{n}\right), \\
&= \Pr\left(m \leq n^{-1}\left(\overline{n}\right)\right), \\
&= \frac{F\left(n^{-1}\left(\overline{n}\right)\right) - F\left(z_2\right)}{1 - F\left(z_2\right)}.
\end{aligned}$$

Then

$$\begin{aligned}
\frac{f\left(m\left(n\right)\right)}{1 - F\left(z_2\right)}\left(\frac{dm}{dn}\right) &= \frac{\frac{\lambda \exp[-\lambda m(n)]}{\exp[-\lambda L]-\exp[-\lambda H]}}{\frac{\exp[-\lambda z_2]-\exp[-\lambda H]}{\exp[-\lambda L]-\exp[-\lambda H]}}\left(\frac{dm}{dn}\right) \\
&= \frac{\lambda \exp\left[-\lambda m\left(n\right)\right]}{\exp\left[-\lambda z_2\right] - \exp\left[-\lambda H\right]}\left(\frac{dm}{dn}\right).
\end{aligned}$$

We can then write the firm size density as

$$r\left(n\right) = B \times n^{-q-1},$$

where

$$B = \exp\left[\left(-\lambda\right)\left[\left[H - L\right] a^{\alpha}\left(\frac{k_1}{k_4^2}\alpha \ln a - \frac{k_1}{k_4^2}k_3 + \frac{k_2}{k_4}\right) + L\right]\right],$$

$$q = \lambda\left[H - L\right] a^{\alpha}\frac{k_1}{k_4^2}\alpha.$$