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A quantitative theory of tax evasion☆

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ABSTRACT

I present a simple, unified approach to study the tax evasion practices often observed in developing countries. I develop a general equilibrium model where heterogeneous establishments optimally select themselves into informality, tax compliance, and formal tax evasion. Informal firms evade taxes by staying small, while larger, formal firms can engage in costly tax evasion. In equilibrium, tax revenues rely on medium-sized firms, which are scarce. In a calibration exercise using data from Mexico, I find that reducing the returns to tax evasion by formal firms increases tax revenues by up to 68%. However, economies where such returns are too high face a trade-off between tax collection and aggregate efficiency, as cracking down on formal tax-evading firms pushes some firms into informality. Last, as the economy develops, the informal sector shrinks, while the tax-evading sector expands, thus limiting potential collection. If lower informality is a byproduct of development, and not vice versa, a solid tax base can be achieved by fiscal authorities effectively by focusing on formal tax evasion.

1. Introduction

Tax evasion is pervasive in developing countries. Many firms choose to join the informal sector, remaining small and avoiding taxes altogether, whereas others are able to reduce their tax burden through lawyers, accountants, and bribes or other forms of corruption. While there are numerous studies that model informality and its aggregate effects, the rent-seeking activities undertaken by many formal firms are largely ignored in modern theories of production and firm heterogeneity.

Tax evasion and tax avoidance have always existed: from wealthy Romans in the third century burying their jewelry to avoid the luxury tax, to eighteen-century English homeowners who bricked up their fireplaces to escape notice from the hearth tax collector (Slemrod, 2007, on Webber and Wildavsky, 1986), to Apple's multi-billion dollar accounts in offshore tax havens.[1] Even in the modern-day U.S., Slemrod (2007) cites an IRS estimate of 17% for the noncompliance rate of the

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[1] See "Apple's Web of Tax Shelters Saved It Billions, Panel Finds," *The New York Times*, May 20, 2013.

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corporate income tax in 2001. The World Bank (2015b) estimates that 54% of companies across 135 developing countries do not report all income tax to authorities, while Artavanis et al. (2015) report a not-so-small figure of 36% for Europe.

Evidently, these tax-evading activities are not costless—the wealthy Romans and the English homeowners spent some of their valuable time digging and laying bricks, while Apple undoubtedly hires many skilled accountants and lawyers to devise and execute their tax-minimizing strategies. Also implied in these anecdotes is the notion that higher stakes usually command higher efforts, as hiding personal jewelry certainly requires less resources than avoiding a multi-billion dollar tax bill. The idea that larger, more productive firms find it more attractive to engage in "defensive" rent seeking is also recognized by Tullock (1992).

In this paper, I propose a simple theory of how agents optimally choose the tax-evading efforts just described as a function of their productivity, market prices, and the institutional environment. Specifically, I consider an environment where formal firms can reduce their fiscal burden by spending resources—either legally or illegally. This formulation is not particular to developing countries, and can be thought of as a quantitative framework to think about the ideas first posed by Krueger (1974). The degree of rent seeking observed in the model economy depends on the stringency of the tax system, as well as on the returns to firms' tax-evading efforts. The theory predicts that larger, more productive firms spend more resources in tax-avoiding/evading activities, and thus face a lower tax burden.

I apply the theory to the specific case of business income taxes, and show how the mixture of formal tax-evading, formal tax-compliant, and informal firms is determined in equilibrium. To this end, I consider a general equilibrium environment where individuals with idiosyncratic managerial abilities face the choice of becoming formal entrepreneurs, informal entrepreneurs, or workers. Informal entrepreneurs avoid paying taxes by staying small, while formal entrepreneurs have the choice of complying with the tax code, or spending resources to reduce their fiscal outlay. The coexistence of small informal firms that do not pay taxes and large formal tax-evading firms results in an effective tax schedule that relies on medium-sized firms. This result links the evidence on the "missing middle" of the distribution of firm sizes in developing countries to a low capacity of the state to generate tax revenues—which is another common feature of many developing countries.

I then study the aggregate effects reducing the returns to the tax-evading efforts undertaken by formal firms. I show that the effects depend on whether there are formal tax-compliant firms operating before any policy changes. Specifically, if there are no formal tax-compliant firms operating, reducing the effectiveness of tax evasion by formal firms pushes previously formal firms into informality, increasing the size of the informal sector, reducing TFP, capital, and output. However, in economies where the returns to tax evasion are not too high, reducing such returns increases the share of tax compliant firms, increasing tax revenues without distorting the allocation of capital and labor.

I calibrate the model to the Mexican economy—where informal firms employ 34% of workers, and tax evasion by formal firms is estimated at 37% of tax collection. I then perform a numerical exploration of the effects of the returns to tax evasion and the statutory tax rate. I find that reducing formal tax evasion increases tax revenues as a percentage of GDP by up to 68%. Further, the Laffer curve with respect to the statutory tax rate generated by the model suggests that Mexico's recent income tax rates have been near the revenue-maximizing value, and thus the state's capacity to raise revenues is unlikely to improve via further changes to these rates.

Finally, I explore the effects of economic development on the equilibrium mixture of firms, tax revenues, as well as the amount of tax evasion in the economy. As the economy grows, the informal sector shrinks, and eventually disappears. However, the share of formal, tax-evading firms continues to grow with the economy, limiting the state's capacity to raise tax revenues even as the economy prospers. These results suggests that, to the extent that lower informality is a byproduct of development, and not vice versa, a solid tax base can be achieved by fiscal authorities more effectively by targeting formal, tax-evading firms.

Clearly, there are other aspects of formal tax evasion and tax avoidance that deserve attention. The feature of tax evasion often highlighted in the literature is the probability of being caught by the tax authorities, which usually comes with a punishment, as in the seminal work of Allingham and Sandmo (1972). In this paper, I focus on a largely unexplored dimension of firm-level tax evasion and tax avoidance, namely, that they are costly activities optimally chosen by firms, much like any other productive input, and that they are the result of the institutional environment, such as loop holes in the tax code, and limits to the state's capacity to enforce it. My treatment of tax avoidance and tax evasion is similar to Mayshar (1991), and Slemrod (2001), who study the partial equilibrium decision of a utility-maximizing individual taxpayer with access to a "tax technology" that allows him to exert labor effort to reduce his tax burden. Acemoglu (2005) and Piketty et al. (2014) also consider economies where tax sheltering is costly, although their focus is on a different set of issues.

My treatment of informality borrows heavily from Fortin et al. (1997), and Leal-Ordóñez (2014), who study the aggregate effects of informality due to incomplete tax enforcement at the extensive margin. In those models, however, formal status implies full compliance, and all formal firms pay the statutory tax rate. One of the contributions of my work is to complement their analysis by considering the effects of incomplete tax compliance at the intensive margin, which is the result of the tax-evading efforts undertaken by formal firms, and to study how both margins interact in equilibrium.[2] By embedding it into a general equilibrium environment with heterogenous firms, this unified approach also contributes to the existing literature on tax evasion using representative agent models, such as Chen and Been-Lon (2003). Moreover, my model gen-

[2] In the same spirit, Ulyssea (2014) considers an economy where firms can be informal either by not registering their business, or by hiring workers "off the books.". Even though I consider ways in which firms avoid taxes different than outsourcing of employees, to the extent that outsourcing is a costly activity that brings the firm some tax benefits, Ulyssea's core idea is implicit in my formulation.

erates optimal tax-evading efforts—with their corresponding effective tax rates—that depend not only on the institutional environment, as in Chen and Been-Lon (2003), but also on the size of the enterprise, thus capturing a realistic aspect of tax evasion suggested by the historical examples—as well as the earlier theoretical work—previously highlighted.

2. A model economy

2.1. Entrepreneurial choice and tax compliance status

There is a representative household populated by a unit continuum of members. Entrepreneurial ability is distributed over the household members according to some distribution $F(Z)$, with bounded support $[Z_L, Z_H]$, and pdf $f(Z)$; with $Z_L \geq 0$. Agents can choose to become formal entrepreneurs, informal entrepreneurs, or workers. Formal entrepreneurs have, in turn, the choice of being fully compliant with the tax code, or to engage in costly rent-seeking activities to reduce their tax burden.

If agents choose to become entrepreneurs—irrespective of the type—they can operate a diminishing-returns to scale technology that utilizes capital and labor as inputs to produce a homogeneous good, as in Lucas and Robert (1978). Output by an entrepreneur of ability Z is given by

$$Y = Z^{1-\theta}\left(K^{\alpha}N^{1-\alpha}\right)^{\theta}, \tag{1}$$

where K and N are the amount of capital and the number of workers hired by the firm, $\alpha \in (0, 1)$ is the share of capital and $\theta \in (0, 1)$ measures the span of control. If the agent chooses to become a worker, he earns a wage w.

Firms face a statutory tax on profits τ_0. Formal firms can choose to comply with the statutory tax, or to pay bribes or hire expediters—accountants, lawyers, or other facilitators—to reduce their tax liability. I call these rent-seeking expenditures B.

In the derivations below, I assume the tax-evasion technology faced by a formal firm takes the following functional form:

$$\tau(B) = \tau_0 \exp(-\phi B),$$

where τ is the effective tax rate, and $\phi \geq 0$ measures the returns to the formal entrepreneur's expenditures on tax evasion, B. The returns parameter has the natural interpretation of a pseudo-elasticity of the effective tax rate with respect to tax-evasion expenditures, since $\partial \ln \tau(B)/\partial B = -\phi$. In general, the pair (τ_0, ϕ) can vary across countries, across sectors, or even across firms. In what follows I assume τ_0 and ϕ are the same for all firms. The specific functional form makes the model highly tractable, and has a number of desirable properties:

i. $\tau'(B) = -\phi\tau_0 \exp(-\phi B) < 0$.
ii. $\tau''(B) = -\phi\tau'(B) > 0$.
iii. $\tau(0) = \tau_0$.
iv. $\lim_{B\to\infty} \tau(B) = 0$.
v. $\lim_{B\to\infty} \tau'(B) = 0$.
vi. $\lim_{B\to 0} \tau'(B) = -\phi\tau_0 < 0$.

The first two properties of the tax-evasion technology are the same as those proposed by Mayshar (1991), and Slemrod (2001), namely, the effective tax rate is decreasing and convex in the tax-evading effort. The rest of the properties provide more structure to the problem: if firms choose not to spend resources on tax evasion, then their effective tax rate is the statutory rate. As the amount of resources spent on tax evasion becomes arbitrarily large, the effective tax rate approaches zero, but so the marginal returns to those expenditures. Thus, an internal solution for B implies an effective tax rate that is lower than the statutory tax rate, but strictly positive.[3] Last, there are marginal gains from engaging in tax evasion when $B = 0$, but they are bounded. Therefore, a corner solution with $B = 0$ is possible.

In general, tax evasion is a gamble not explicitly considered in this formulation. However, the risky aspect of tax evasion can be presented in this framework if, as explained by Mayshar (1991), one defines B as the payment which generates the same expected profit loss as the extra risk an evading firm takes on, for given expected tax payments. In what follows I use the terms "evasion" and "avoidance" interchangeably.

If the entrepreneur chooses to evade taxes, his problem is then to choose (K_E, N_E, B), given Z, τ_0, ϕ and factor prices (w, r), to maximize profits,

$$\Pi_E(Z) \equiv \max_{K_E, N_E, B} \left(1 - \tau_0 e^{-\phi B}\right)\left[Z^{1-\theta}\left(K_E^{\alpha}N_E^{1-\alpha}\right)^{\theta} - rK_E - wN_E\right] - B,$$

subject to $B \geq 0$.

The optimal choices by the entrepreneur are

$$K_E(Z) = Z\left[\frac{\alpha\theta}{r}\kappa^{-\theta(1-\alpha)}\right]^{\frac{1}{1-\theta}}, \tag{2}$$

[3] Numerically, it is possible to generate effective tax rates that are indistinguishable from zero.

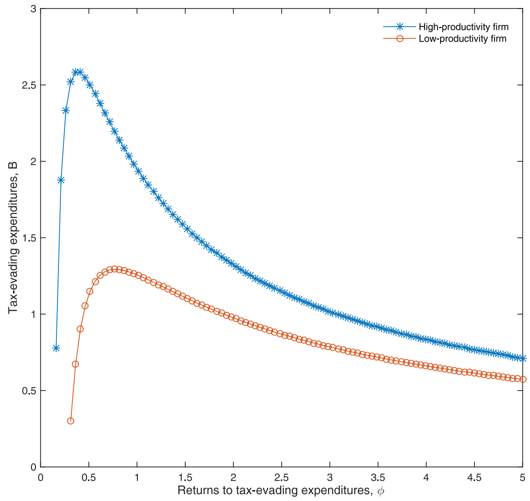

Fig. 1. Tax-evasion expenditures as a function of ϕ.

$$N_E(Z) = Z\left[\frac{(1-\alpha)\theta}{w}\kappa^{\theta\alpha}\right]^{\frac{1}{1-\theta}}, \tag{3}$$

$$B(Z) = \frac{1}{\phi}\ln\left(\tau_0\phi\left[Z^{1-\theta}\left(K_E^{\alpha}N_E^{1-\alpha}\right)^{\theta} - rK_E - wN_E\right]\right), \tag{4}$$

where

$$\kappa \equiv \frac{K_E}{N_E} = \left(\frac{\alpha}{1-\alpha}\right)\frac{w}{r}$$

is the capital-labor ratio common to all formal firms. Notice that in the case of a tax of profits, the input choices are independent of the choice of rent-seeking expenditures and the effective tax rate. Therefore, neither the statutory tax, or the possibility of reducing it, distort the optimal choices of capital and labor. In this economy, larger, more productive and profitable firms spend more in rent seeking and thus have a lower tax burden—a realistic feature highlighted by anecdotal and historical evidence, as well as earlier theories of rent seeking, but missing in more recent representative agent models of tax evasion, such as Chen and Been-Lon (2003). Further, tax evasion is increasing in the statutory tax rate, as in Chen and Been-Lon (2003).

Fig. 1 shows expenditures on tax evasion as a function of the returns parameter, ϕ, for high and low productivity firms. For any level of returns, more productive firms spend more on tax evasion. The main feature to notice, however, is the non-monotonic relationship between expenditures on tax evasion and the returns parameter. Shleiffer et al. (1993) highlight the same non-monotonic feature of bribe payments in a corrupt system. They argue that when the returns to bribing are either too low or too high, we should observe smaller bribes, and thus bribes exhibit a Laffer curve type of property in relation to the returns to bribing.

Let $\Psi(w, r)$ be defined as

$$\Psi(w,r) = (1-\theta)\left(\frac{\theta\alpha}{r}\right)^{\frac{\alpha\theta}{1-\theta}}\left(\frac{\theta(1-\alpha)}{w}\right)^{\frac{(1-\alpha)\theta}{1-\theta}} \tag{5}$$

The profits of the tax-evading entrepreneur are then given by

$$\Pi_E(Z) = Z\Psi(w,r) - \frac{1}{\phi}[1+\log(\tau_0\phi Z\Psi(w,r))]. \tag{6}$$

Entrepreneurs who chose to comply with the tax code solve the following problem

$$\Pi_C(Z) \equiv \max_{K_C,N_C}(1-\tau_0)\left[Z^{1-\theta}\left(K_C^{\alpha}N_C^{1-\alpha}\right)^{\theta} - rK_C - wN_C\right],$$

Their input choices are

$$K_C(Z) = Z\left[\frac{\alpha\theta}{r}\kappa^{-\theta(1-\alpha)}\right]^{\frac{1}{1-\theta}}, \tag{7}$$

$$N_C(Z) = Z\left[\frac{(1-\alpha)\theta}{w}\kappa^{\theta\alpha}\right]^{\frac{1}{1-\theta}}, \tag{8}$$

with profits

$$\Pi_C(Z) = (1-\tau_0)Z\Psi(w,r). \tag{9}$$

I model informality as in Leal-Ordóñez (2014). Informal entrepreneurs do not pay any taxes. They are able to do so by staying small. I assume that the government has the ability to detect any firm with a capital stock greater than some $D > 0$. In the case of detection, the firm is shut down and the entrepreneur earns zero profits. Therefore, the greater D, the lower the ability of the government to detect informal firms. This threshold strategy is consistent with optimal tax enforcement by a government with low resources that is able to observe input choices by firms, which are in turn signals of the entrepreneur's productivity, as shown by Bigio et al. (2011).

Thus, we can write the problem solved by informal entrepreneurs as follows,

$$\Pi_I(Z) \equiv \max_{K_I,N_I} Z^{1-\theta}\left(K_I^{\alpha}N_I^{1-\alpha}\right)^{\theta} - rK_I - wN_I,$$

subject to

$$K_I \le D.$$

Let $\mu(Z)$ denote the multiplier on the size constraint—the shadow cost of informality. Highly productive entrepreneurs will face a higher $\mu(Z)$, while unproductive entrepreneurs may not be bound by the size constraint at all, thus facing a multiplier equal to zero. As long as there is a positive measure of informal entrepreneurs for which $\mu(Z) > 0$, there will be aggregate productivity losses from informality. In Appendix A, I provide a thorough theoretical discussion on how informality affects TFP.

The input choices of informal firms are

$$K_I(Z) = Z\left[\frac{\alpha\theta}{r+\mu(Z)}\tilde{\kappa}(Z)^{-\theta(1-\alpha)}\right]^{\frac{1}{1-\theta}}, \tag{10}$$

$$N_I(Z) = Z\left[\frac{(1-\alpha)\theta}{w}\tilde{\kappa}(Z)^{\theta\alpha}\right]^{\frac{1}{1-\theta}}, \tag{11}$$

where

$$\tilde{\kappa}(Z) = \left(\frac{\alpha}{1-\alpha}\right)\frac{w}{r+\mu(Z)}$$

is the capital-labor ratio, which varies across informal firms. Firms that face a higher shadow cost of informality have a lower capital-labor ratio.

The profits of informal firms are then,

$$\Pi_I(Z) = \omega(Z)Z\Psi(w,r). \tag{12}$$

where $\omega(Z) = \left(\frac{r}{r+\mu(Z)}\right)^{\frac{\alpha\theta}{1-\theta}}$ is the profit wedge caused by firms' choice of staying small to avoid paying taxes, resulting in a fraction $1-\omega(Z)$ of profits being lost—an implicit informality tax. Fig. 2 shows $\omega(Z)$. As Z increases, it becomes less profitable to operate as an informal firm.

With this notion in hand, notice that an entrepreneur's pre-tax (and pre-tax-evasion expenditures when applicable) profits are given by $Z\Psi(w, r)$, regardless of firm type. The differences in net profits across firm types, then, arise from differences in the cost of doing business: informal firms pay an implicit informality tax, formal compliant firms pay the statutory tax, while formal tax-evading firms pay a combination of taxes and tax-evasion expenditures.

Lemma 1. *There exists a unique value Z_E, such that if $Z > Z_E$, formal firms choose to evade taxes. This value is given by $Z_E = [\phi\tau_0\Psi(w,r)]^{-1}$.*

All proofs are in Appendix B. The equilibrium cutoff Z_E is decreasing in ϕ and τ_0. Fig. 3 shows how different combinations of ϕ and Z determine a firm's decision to choose to evade taxes or to comply. When ϕ is very low, only the most productive firms find it profitable to engage in tax evasion. As ϕ increases, the required level of Z to profit from tax evasion decreases.

In the same spirit as the choice of tax evasion by formal firms, the choice of informality will be characterized by a threshold for productivity, which I call Z_I. Entrepreneurs with ability below Z_I will be informal, while those with $Z \ge Z_I$ will be formal. If $Z_I > Z_E$, there are no formal firms that are fully complaint with the tax code: only informal and tax-evading formal firms co-exist. The mixture of firm types will depend on the policy parameters τ_0, ϕ, and D.

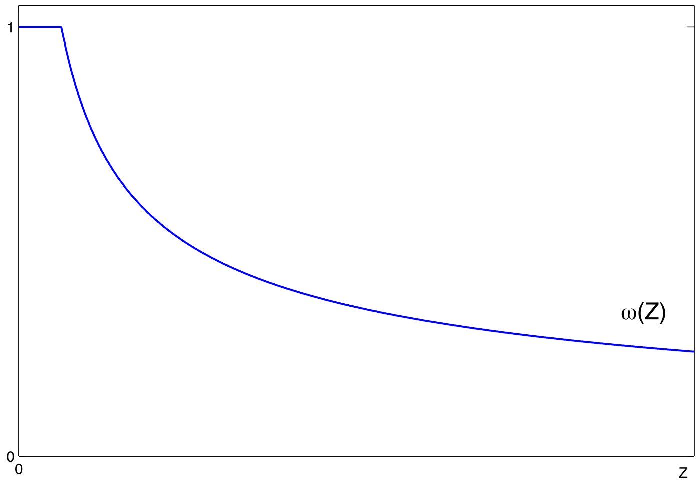

Fig. 2. Profit wedge due to informality.

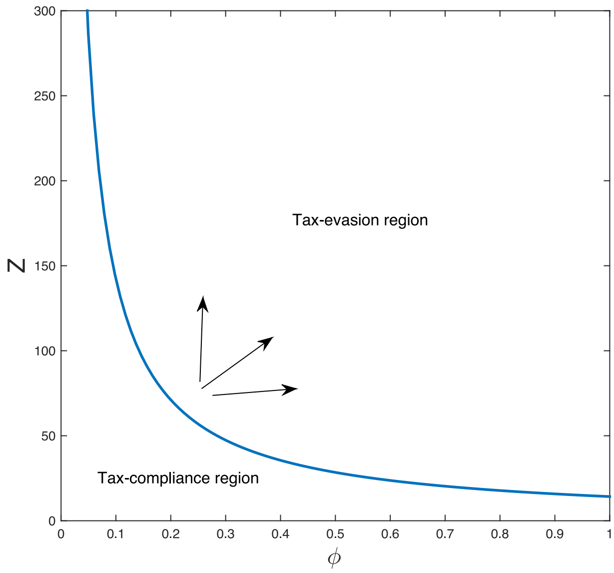

Fig. 3. Tax evasion and tax compliance in the (ϕ, Z) space.

Lemma 2. *If there is a productivity level $Z_P > 0$, such that $\omega(Z_P) > (1-\tau_0)$, then there exists a unique value $Z_I \geq Z_P$, such that $\Pi_I(Z_I) = \max\{\Pi_C(Z_I), \Pi_E(Z_I)\}$. If $Z \leq Z_I$, the entrepreneur chooses to be informal.*

Agents observe their productivity draw Z, and then choose to either become an entrepreneur or a worker, whichever gives them the highest earnings. Their problem is then

$$V(Z) = \max\{\Pi^*(Z; w, r, \phi, \tau_0, D), w\},$$

Where $\Pi^*(Z; w, r, \phi, \tau_0, D) = \max\{\Pi_I(Z), \Pi_C(Z), \Pi_E(Z)\}$.

Lemma 3. *There exists a unique value Z_W such that $V(Z_W) = \Pi^*(Z_W; w, r, \phi, \tau_0, D) = w$. If $Z < Z_W$, the agent chooses to become a worker.*

2.2. Accumulation

The household derives utility from consumption streams only, and discounts the future at a rate $\beta \in (0, 1)$. It is endowed with an initial capital stock $K_0 > 0$, as well as a unit of time each period, which is supplied inelastically. The household consumes and accumulates capital so as to maximize lifetime utility,

$$\max_{C_t, K_{t+1}} \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$C_t + K_{t+1} \leq (1 - \delta + r_t)K_t + \int_{Z_L}^{Z_H} V(Z)dF(Z) + T_t,$$

where $\int_{Z_L}^{Z_H} V(Z)dF(Z)$ are the aggregate household earnings, and T_t is a lump-sum transfer from the government. The function $u(\cdot)$ is strictly increasing and strictly concave and satisfies Inada conditions. At the steady state, the Euler equation provides the standard result for the rental rate of capital

$$r = \frac{1}{\beta} - (1 - \delta).$$

2.3. Government

The government collects revenues R_t from taxes and informality penalties. Since in equilibrium no informal firm is caught, all revenues come from tax collection. I assume the government runs a period-by-period balanced budget, so $R_t = T_t$, all t. Revenues from formal, compliant firms—when these exist—are given by

$$R^C = \int_{Z_I}^{Z_E} \tau_0 \Psi(w, r) Z dF(Z) = \tau_0 \Psi(w, r)[F(Z_E) - F(Z_I)].$$

The effective tax paid by formal, tax-evading firms is given by

$$\tau(Z) = \tau_0 \exp[-\phi B(Z)] = \frac{1}{\phi \Psi(w, r) Z}.$$

Notice that it does not depend on τ_0. Then, when $Z_E > Z_I$ revenues from formal, tax-evading firms are

$$R^E = \int_{Z_E}^{Z_H} \tau(Z) \Psi(w, r) Z dF(Z) = \frac{1}{\phi}[F(Z_H) - F(Z_E)].$$

Total revenues are then $R = R^C + R^E$. When $Z_E < Z_I$, there are no formal firms that comply with the tax code, and so all revenues come from tax-evading firms,

$$R = R^E = \frac{1}{\phi}[F(Z_H) - F(Z_I)].$$

2.4. Equilibrium

A steady-state competitive equilibrium consists of constant input prices (w, r), constant aggregate levels of consumption (C) and capital (K), an occupational choice cutoff Z_W, and firm-type choice cutoffs $\{Z_I, Z_E\}$ with their corresponding collections of input policies $\{K^*(Z; w, r), N^*(Z; w, r), B(Z; w, r)\}$ indexed by Z, such that:

i. The representative household problem is solved: $r = 1/\beta - (1 - \delta)$.
ii. The firm-type choices and their corresponding input policies maximize profits, taking (w, r) as given.
iii. The occupational choices maximize household earnings, taking (w, r) as given.
iv. The labor, capital, and goods markets clear.
v. The government budget is balanced.

The rental rate of capital is determined by the inter-temporal problem of the household. The thresholds—whose existence was determined in the previous section—and the wage rate are determined jointly through the labor market clearing condition.[4]

Fig. 4 shows the equilibrium determination of occupational, and firm type choices for a case in which $Z_E > Z_I$. The figure plots the equilibrium profits for each type of firm, as well as the wage rate, as a function of rescaled productivities $Z^{1-\theta}$.[5] The lower bound for productivities is set at one. Agents with productivities between one and Z_W choose to become workers, whereas those with productivities greater than Z_W but less than Z_I choose to operate informal firms. Informal entrepreneurs with an ability between Z_W and Z_0 are unconstrained by the detection threshold, and thus face a shadow cost of informality, $\mu(Z)$, of zero, while for those with abilities between Z_0 and Z_E, $\mu(Z) > 0$. Entrepreneurs with abilities between Z_I and Z_E choose to comply with the statutory tax rate, whereas those with abilities greater than Z_E engage in costly tax evasion.

[4] The market clearing condition is given by $\int_{Z_L}^{Z_W} dF(Z) = \int_{Z_W}^{Z_H} N^*(Z; w, r) dF(Z)$. Notice that the left-hand side is the household's labor supply, which is strictly increasing in w (since Z_W is increasing in w), while the right-hand side is the aggregate labor demand, which strictly decreasing in w (since Z_W is increasing in w and $N^*(Z; w, r)$ is decreasing in w). Therefore, if an equilibrium exists, it is unique. In fact, since $N^*(Z; w, r) \to 0$ as $w \to \infty$, and $N^*(Z; w, r) \to \infty$ as $w \to 0$, and labor supply is zero when $w = 0$, existence is guaranteed.

[5] The rescaling is just for ease of display purposes.

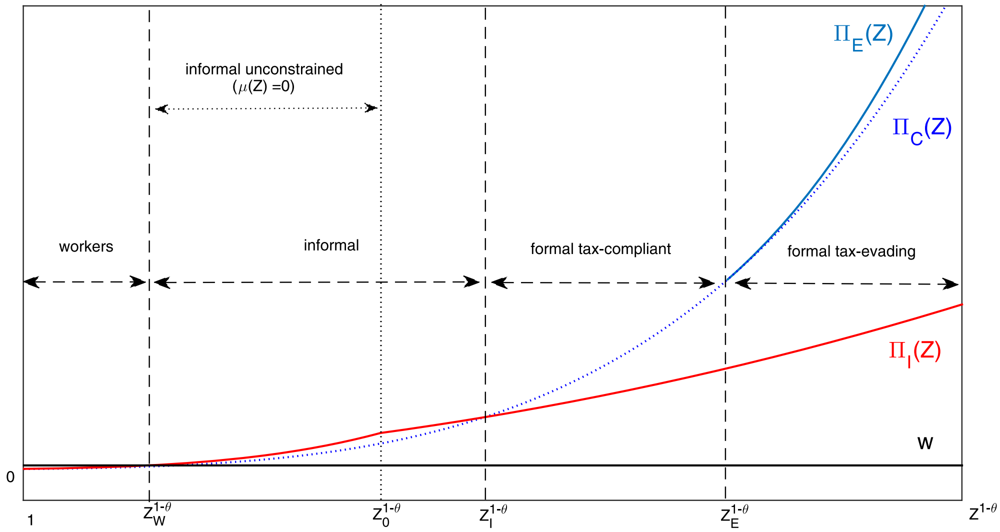

Fig. 4. Equilibrium occupational and firm type choice—Case I: $Z_I < Z_E$.

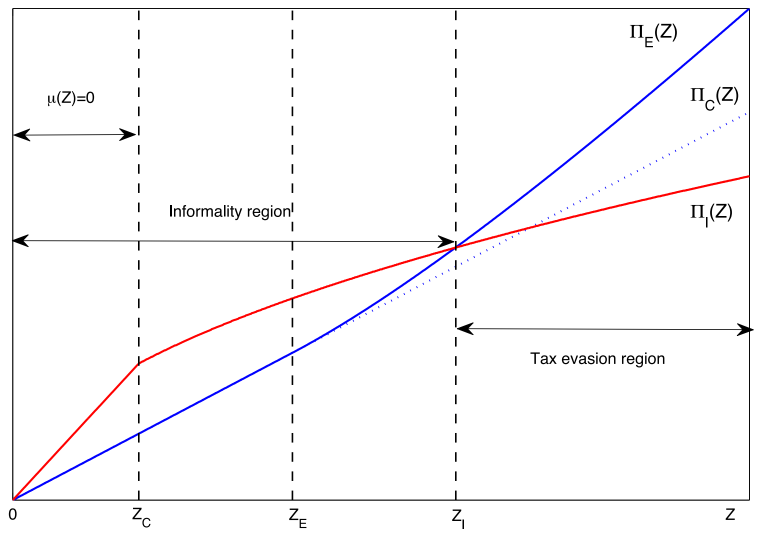

Fig. 5. Equilibrium firm type choice when $Z_I < Z_E$.

2.5. Reducing tax evasion by formal firms

I start with the case in which $Z_I < Z_E$, so that some formal firms choose to comply with the statutory tax. Reducing tax evasion by formal firms can be achieved through a reduction in the returns to tax-evasion expenditures, ϕ. In this case, $Z_E = [\phi\tau_0\Psi(w, r)]^{-1}$ increases, since the change in ϕ does not affect the aggregate demand for labor, and therefore has no effect on w. The result is higher government revenues, and less tax evasion by formal firms. No other aggregate variables are influenced by the reduction in ϕ.

When the economy is at an equilibrium in which $Z_I > Z_E$, only informal and formal tax-evading firms operate. Fig. 5 shows the firm-type choice in this case. A reduction in ϕ decreases the profits of formal firms, $\Pi_E(Z)$, which pushes the informality cutoff Z_I to the right, thus increasing the size of the informal sector. To the extent that there were some constrained informal firms operating before the change, the marginal firms coming into the informal sector as a result of the change in ϕ will be constrained, which will result in lower TFP and a lower aggregate capital stock. In other words, when there are only informal and tax-evading formal firms, lowering the returns to tax evasion has the unintended consequence of pushing firms into informality, thus decreasing aggregate productivity, capital, and output.

The effect on government revenues is not as clear: on one hand, firms that were paying some taxes before the change will not pay any taxes, since they will move to the informal sector. On the other hand, the remaining tax-evading firms will face a stronger enforcement, which increases the revenue collected from them. The total impact on revenues depends on which effect dominates.

Ultimately, changes in either ϕ or τ_0 affect the equilibrium productivity thresholds for firm type and occupational choices, and so their aggregate effects depend on the distribution of productivities, $F(\cdot)$. Thus, to move forward in the discussion of the effects of enforcement, we need to impose a parametric structure and assign parameter values that closely match some aspects of a real economy.

3. Parameterization and calibration

I follow the vast literature on firm-size distributions starting with Axtell (2001), and assume that productivities are distributed according to a Pareto distribution. In particular, I assume that the rescaled managerial ability $Z^{1-\theta}$ satisfies

$$Pr(Z^{1-\theta} \leq z) = \frac{1 - \left(\frac{Z_L^{1-\theta}}{z}\right)^S}{1 - \left(\frac{Z_L^{1-\theta}}{Z_H^{1-\theta}}\right)^S},$$

All parameters are chosen to match certain aspects of the Mexican economy. As explained by Leal-Ordóñez (2014), the lower bound Z_L, can be chosen arbitrarily, since it has to be that $Z_W > Z_L$ for the problem to make sense. That is, in equilibrium, all agents with $Z < Z_W$ become workers.

According to the 2009 Economic Census, firm sizes of Mexican establishments ranged from 1 to 12,226, with an average firm size of 5.5. I calibrate the upper bound Z_H, as well as the shape parameter S, so that the equilibrium distribution of firm sizes matches the average size, and

$$\left(\frac{N_E(Z_H)}{N_I(Z_W)}\right)^{1-\theta} = \left(\frac{Z_H}{\omega_N(Z_W)Z_W}\right)^{1-\theta} = (12,226)^{1-\theta} = 9.5.$$

I calibrate the threshold parameter D to match the share of workers in the informal sector in Mexico, which is 34% according to the International Labor Organization (ILO). It is important to distinguish between "workers employed in the informal sector" and "informal workers." The informal sector is a firm-based concept, encompassing all persons working in productive units that have informal characteristics, including legal status, registration, size, registration of employees and bookkeeping practices. Informal employment, in turn, is a job-based concept, which includes all workers in the formal sector, informal sector, or households, whose main jobs lack basic social or legal protections, or employment benefits (ILO, 2012). It follows that the workers in the informal sector represent a subset of informal employment. I focus on the former, because my model only makes predictions about those workers employed in the informal sector. Total informal workers represent 53% of the labor force, so around a third of them work outside the informal sector. In a recent paper, Ulyssea (2014) shows how both concepts of informality interact in an economy with heterogeneous establishments.

The type of tax I consider in the model most closely resembles a tax on business profits.[6] I set $\tau_0 = 0.3$, which corresponds to the statutory income tax rate for businesses and corporations, and calibrate ϕ so as to match the revenues from business income taxes as a percentage of GDP. According to the Mexican tax authorities (SHCP by its Spanish acronym), business income taxes and other taxes on business profits accounted for 54% of all income tax collected in 2009 (SHCP, 2010). In turn, total income tax revenues for the same year accounted for 4.9% of GDP (OECD, 2015). This implies that the income tax collected from businesses represents 2.7% of GDP.

Tax evasion is, by its secretive nature, hard to measure. In the most recent study commissioned by the tax authorities in Mexico, Fuentes et al. (2010) estimate the average income tax evasion by businesses at one percent of GDP, for the period 2001–2008. This figure amounts to 37% of current tax collection. On the other hand, the official budget for business income tax expense—non-accrued tax revenues due to differential tax rates, exemptions, subsidies and tax credits, fiscal stimulus, authorized deductions, and special tax regimes and treatments—represented 1.2% of GDP in 2009 (SHCP, 2009). Because these figures on tax evasion and government tax expense are imperfect estimates, I do not rely on them for the baseline calibration.[7]

The discount rate β is set to match the capital-output ratio. I use the calculations of Restuccia (2008) and Leal-Ordóñez (2014), who find values for the capital-output ratio of 1.9 and 2 for Mexico, using distinct data sources. The depreciation rate δ is taken from INEGI's own calculation using the 2014 Economic Census (INEGI, 2014). The rest of the parameters are taken from related papers that use Mexican data: I take the capital share of income α from Bergoeing et al. (2002), and the diminishing returns parameter θ from Leal-Ordóñez (2014). All sources and target moments are listed in Table 1.

Table 1
Parameter values.

Parameter	Definition	Value	Source
Assigned			
α	Capital share	0.3	Bergoeing et al. (2002)
δ	Depreciation rate	0.07	INEGI (2014) Economic Census
τ_0	Statutory tax rate	0.3	Mexico's business income tax rate
θ	Span of control	0.76	Leal-Ordóñez (2014)
$Z_L^{1-\theta}$	Pareto lower bound	1	Arbitrary
Calibrated (jointly)			*Target moments*
D	Informality detection	6.4407	Size of informal sector (emp.)
ϕ	Returns to tax evasion	1.0897	Revenues/GDP
$Z_H^{1-\theta}$	Pareto upper bound	12.5869	Size range
s	Pareto shape	6.7956	Average firm size
β	Discount rate	0.9578	Capital-output ratio

Table 2
Calibration results: moment matching.

Targeted	Data	Model
Size of informal sector (% of emp.)	0.34	0.35
Revenues (% of GDP)	0.027	0.028
$\left(\frac{N_{max}}{N_{min}}\right)^{1-\theta}$	9.5	9.5
Average firm size	5.5	5.5
Capital-output ratio	1.9	1.9
Non-targeted	Data	Model
Share of firms of size $<= 10$	0.94	0.91
Share of firms of size between 11 and 50	0.05	0.08
Share of firms of size between 51 and 250	0.008	0.006
Share of firms of size > 250	0.002	0.004

Table 2 shows the calibration results.[8] The model successfully matches all targeted moments, and does a decent job at matching some non-targeted moments.[9] Most firms in Mexico are small and informal, which is also the case in the baseline calibration. However, the model predicts a share of medium sized firms that is larger than that observed in the data (8.07% vs. 4.65%).

I use financial statement data from COMPUSTAT Global, and estimate average effective tax rates (ETR) for an unbalanced panel of public firms in Mexico for the period 1990–2014. I compute ETR as the ratio of total income tax expense (TXT) to the difference between pre-tax income (PI) and special items (SPI). McGuire et al. (2012) use the same definition of ETR to study the effects of industry-specific experience of managers on tax avoidance. These ETRs are a good, if imperfect, candidate for the empirical counterpart of the effective tax rates in the model. For instance, Mills (1998) suggest that expenditures on tax planning result in lower ETRs, whereas (Cook et al., 2008) provide evidence that the magnitude of the fees paid to external auditors is associated with greater reductions in ETRs.

ETRs in the sample exhibit large cross-sectional and time-series variability. The time-series variability stems in part from the ability that firms have to spread their tax bill over time, often deferring tax payments due in the current year to future years. I try to address this issue not considered by the theory by taking the time-series average for each firm's ETR in the sample, dropping those firms with less than five years of data available. I then focus on two subsamples: one comprised of firms with an average ETR between 0 and 70%, as in McGuire et al. (2012), and a second with firms with an average ETR

[6] In Appendix C, I calibrate a version of the model that considers a tax on output—as opposed to a tax on profits—which creates distortions from tax evasion in both partial and general equilibrium settings.

[7] However, they are useful as benchmarks to compare counterfactual results.

[8] The numerical solution to the general equilibrium problem amounts to solving a standard excess demand function for the labor market. The existence and uniqueness properties of the equilibrium are described in the previous section. It takes MATLAB about one second to solve this problem for given parameter values.The joint calibration consists of choosing the parameter values that minimize the mean square error (MSE) between the moments from the data, and those generated by the model. This can be accomplished with standard grid search algorithms built into MATLAB, such as *fminsearch*. The algorithm solves the model for a given set of parameter values, computes the moments to be matched, and then compares them to those in the data. If the MSE is small enough, it stops, otherwise it continues searching.

[9] With the exception of the observation for the size of the largest establishment, which comes from restricted-access data from the 2009 Economic Census, all data on the size distribution of establishments come from publicly available reports by INEGI, using data from the 2014 Economic Census. The moments of the size distribution considered in the baseline calibration and in the tests of the model remained virtually unchanged from the 2009 to the 2014 Economic Census.

Table 3
Effective tax rates.

	Median	Mean	S.D.
Data—0-70% sample	0.26	0.25	0.12
Data—0-35% sample	0.24	0.22	0.09
Model—All formal firms	0.30	0.26	0.07
Model—Tax-evading firms	0.20	0.19	0.08

Data source: Author's calculations using COMPUSTAT Global.

Table 4
Equilibrium mixture of firms—baseline calibration.

Firm type	Share of total firms	Share of total employment
Informal	0.71	0.34
Tax compliant	0.19	0.22
Tax evading	0.10	0.44

between 0 and 35%, which is the highest statutory income tax rate during the sample period. The resulting sample sizes are 134 and 114.[10] Half the firms in both samples have at least 15 years of data available.

The ETR data are far from representative of formal firms in Mexico, as only 0.1% of all firms are publicly listed. Table 3 compares the total median, mean and standard deviations of the calculated ETRs, with those predicted by the baseline calibration for all formal firms, and for formal, tax-evading firms. The mean and median estimates from both COMPUSTAT subsamples lie somewhere in between their model counterparts.

The model generates lower variability in ETRs than that observed even in the more restricted sample. This occurs because the model cannot generate either an ETR above the statutory tax rate, or the amount of firms with an ETR of zero that are present in the data. Even though I have partially adjusted for the ability of firms to shuffle their tax burden over time, the time horizon for any given firm is not long enough to completely account for this aspect of reality.

The inability of the model to produce a larger share of ETRs with a value of zero arises, in part, because the theory predicts a one-to-one, negative relationship between ETR and productivity, which may not be the case in the data. Further, the distribution of productivities was constructed to match the size distribution of establishments in Mexico, which has a thin right tail. Thus, in the baseline calibration, there are as many instances of zero ETRs as there are very large establishments. Given the lack of representativeness of the COMPUSTAT data, one should not expect the model—which is calibrated to match aspects of all firms—to match the observed distribution of ETRs.

Table 4 shows the equilibrium mixture of establishments in the baseline calibration, where 71% of all firms are informal. This figure is smaller than the 87% reported by Hsieh and Klenow (2014) for manufacturing firms in Mexico, using the Economic Census from 2009. Tax compliant firms account for 19% of all firms and 22% of total employment, while tax evading firms represent 10% of all firms and 44% of employment.

The coexistence of small informal firms that pay no taxes with large formal tax-evading firms gives rise to an effective tax schedule that relies on medium-sized firms. In a model where tax authorities are optimizing agents that choose auditing schemes based on observable input choices by firms, Zilberman (2016) also finds this non-monotonicity of effective tax rates with respect to productivity. Medium-sized firms account for a disproportionately large share of total tax revenues. In the baseline calibration, firms of sizes 5–50 account for 94% of total revenues. This result ties the "missing middle" phenomenon—the scarcity of small and medium-sized firms relative to microenterprises and large firms—to a low capacity to generate tax revenues. Hsieh and Olken (2014) have recently shown that, in reality, both middle and large firms are missing from the distribution of firm sizes in developing countries. Mexico is, in fact, an example where both medium and large firms are scarce relative to micro firms: 75% of firms have four or less employees and, as shown in Table 2, the right tail is very thin. Still, the model links either interpretation of the "missing middle" to low income tax revenues.

4. Numerical exercises and discussion

In this section, I study how different values of the returns parameter, ϕ, and the statutory tax rate, τ_0, affect the equilibrium values of several aggregates of interest, keeping the rest of the parameters unchanged from the baseline calibration. Table 5 shows results for different values of ϕ, ranging from zero, in column (1), to 50, in column (7). Results from the baseline calibration are presented in column (3).

To better illustrate one of the more subtle, yet important, points of the theory—namely, that when there are no formal tax-compliant firms, tax evasion is distortionary—we can start from column (7) in Table 5—when ϕ is very high—and see how aggregate variables change as ϕ decreases. When $\phi = 50$ there are no tax-compliant firms: one third of firms are

[10] As of October 2015, there are 140 firms listed in the Mexican Stock Exchange.

Table 5
Returns to tax evasion, extreme values.

Aggregate	(1) $\phi = 0$	(2) $\phi = 0.55$	(3) $\phi = 1.09$	(4) $\phi = 2$	(5) $\phi = 5$	(6) $\phi = 10$	(7) $\phi = 50$
TFP*	1.000	1.000	1.000	1.000	1.005	1.006	1.009
K-stock*	1.000	1.000	1.000	1.000	1.010	1.016	1.024
Output*	1.000	1.000	1.000	1.000	1.023	1.044	1.069
Wage*	1.000	1.000	1.000	1.000	1.002	1.008	1.011
Informal sector (% of employment)	0.349	0.349	0.349	0.349	0.309	0.248	0.143
Tax revenues (% of GDP)	0.047	0.035	0.028	0.019	0.009	0.006	0.002
Share of tax-compliant firms	0.286	0.254	0.187	0.027	0.000	0.000	0.000
Share of informal firms	0.714	0.714	0.714	0.714	0.650	0.546	0.334
Share of tax-evading firms	0.000	0.032	0.099	0.259	0.350	0.454	0.666
Rent-seeking (% of GDP)	0.000	0.005	0.007	0.011	0.013	0.011	0.006

*Values relative to baseline. $\phi = 1.0897$ in baseline calibration.

Table 6
Alternative statutory tax rates.

Aggregate	(1) $\tau_0 = 0.15$	(2) $\tau_0 = 0.2$	(3) $\tau_0 = 0.25$	(4) $\tau_0 = 0.3$	(5) $\tau_0 = 0.35$	(6) $\tau_0 = 0.4$
TFP*	1.006	1.006	1.005	1.000	0.966	0.962
K-stock*	1.018	1.015	1.010	1.000	0.944	0.934
Output*	1.055	1.040	1.023	1.000	0.928	0.903
Wage*	1.010	1.007	1.002	1.000	1.000	0.995
Informal sector (% of employment)	0.206	0.261	0.309	0.349	0.425	0.461
Tax revenues (% of GDP)	0.023	0.026	0.028	0.028	0.024	0.021
Share of tax-compliant firms	0.503	0.381	0.279	0.187	0.080	0.007
Share of informal firms	0.466	0.569	0.650	0.714	0.799	0.841
Share of tax-evading firms	0.031	0.050	0.071	0.099	0.122	0.152
Rent-seeking (% of GDP)	0.002	0.004	0.005	0.007	0.010	0.012

*Values relative to baseline. $\tau_0 = 0.3$ in baseline calibration.

informal, and two thirds are formal, tax-evading firms. As ϕ decreases, previously formal tax-evading firms leave the formal sector entirely—those whose productivity is not high enough to find formal tax evasion profitable. Some of these informal firms will be constrained by the detection threshold—which remains unchanged throughout these exercises—causing TFP, capital, and output to decline. As ϕ decreases further, some firms choose compliance over informality. These are medium-sized firms: they are not productive enough to find formal tax evasion appealing at the lower return, but they are too productive to join the informal sector. Notice that for values of ϕ below 2, tax evasion no longer has an effect on the size of the informal sector, TFP, capital, or output. All the gains in tax revenues in that range come from previously formal tax-evading firms becoming compliant. The baseline value of ϕ falls within that range.

Table 6 contains the results from solving the model for different statutory tax rates, starting at 15%, up to 40%. The results using the baseline value of $\tau_0 = 0.3$ are in column (4). A lower tax rate increases TFP, capital, and output because it lowers the cost of formality. For lower values of τ_0, compliance becomes more attractive not only for informal firms, but also for formal, tax-evading firms. Tax revenues, however, exhibit a Laffer curve: high tax rates push firms into either informality or formal tax evasion, which reduces the tax base, whereas low tax rates increase the tax base, but decrease the amount collected from each firm. In fact, as shown in Fig. 6, recent income tax rates in Mexico have been near their revenue-maximizing values. Notice that this counterfactual Laffer curve takes into account the general equilibrium responses of both informality and tax evasion by formal firms, generated by changes in the statutory tax rate.

4.1. The cost of collection and the tax base

A natural question to ask is whether increasing tax collection via a reduction in the returns to tax evasion is worth the cost to the tax authorities. The model is silent about this trade off, but we can rely on official estimates on the cost of collection to estimate the potential cost of the increase in revenues predicted by the model. Fig. 7 shows that tax collection in Mexico has been increasing since 2006, while the costs of collection have declined. The model predicts that reducing the returns to tax evasion by half increases tax revenues by 25%, whereas further reducing the returns to zero increases revenues by 68%. Even if we take the highest cost estimate of nearly 1% of tax revenues, the increase in revenues predicted by the model more than compensate for the associated cost of collection.

The recent increase in revenues observed in Mexico has been partially driven by a growing tax base. Fig. 8 shows the size and composition of Mexico's tax base for the period 2011–2016. The three main categories considered by the tax authorities are wage workers, individual non-workers, and companies and corporations. The category "individual non-workers" includes freelancers, and businesses who declare taxes under the owner's name, rather than a company name. Businesses under the "individual non-workers" denomination are more likely to be small and medium-sized enterprises. It is clear from Fig. 8 that

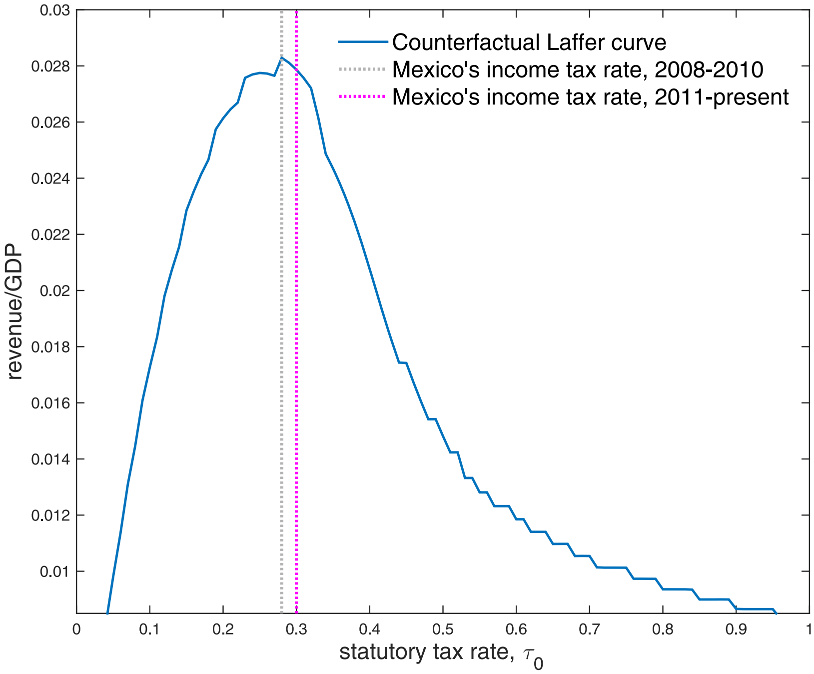

Fig. 6. Counterfactual Laffer curve and recent income tax rates in Mexico.

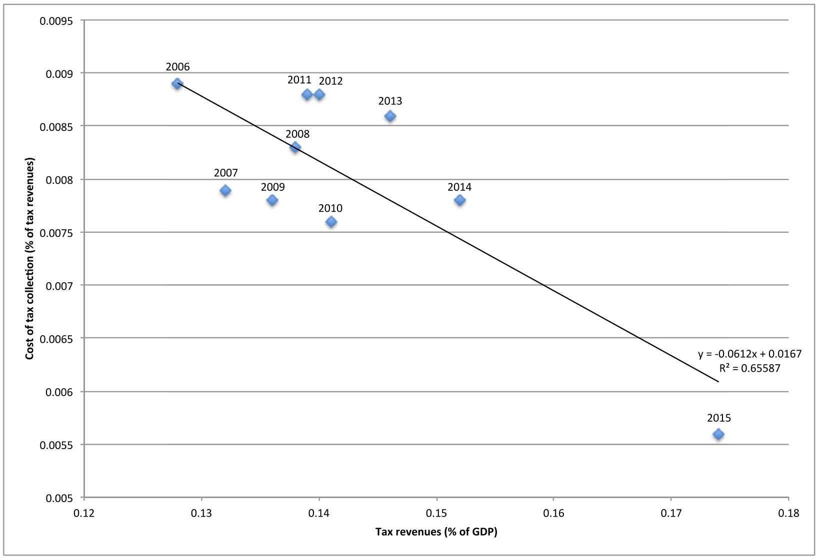

Fig. 7. Tax revenues and the costs of tax collection in Mexico, 2006–2015.

this category is the largest and fastest growing of the tax base, which I consider as suggestive evidence in favor of the model's predictions on how revenues rely on medium-sized firms.

4.2. The effects of economic development

Another question we could ask is: will the composition of firms change, without any policy intervention, as the economy develops? The answer is, not surprisingly, yes. One could carry out such exercise, in a very reduced-form way, by considering

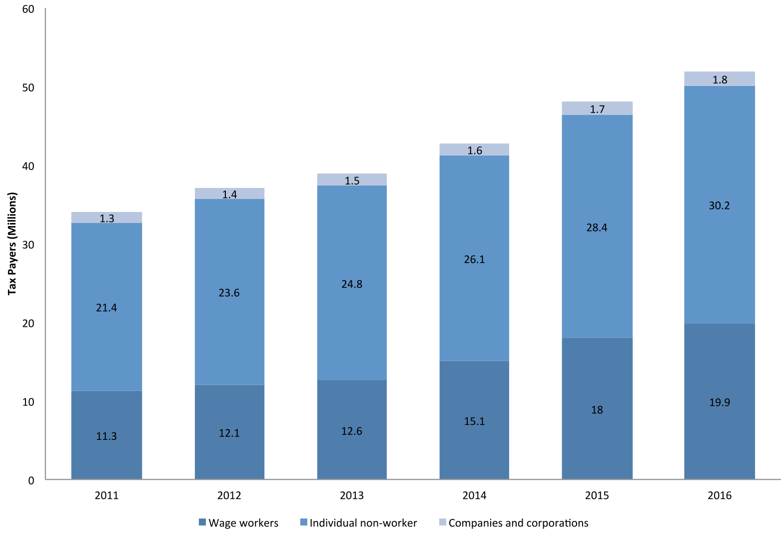

Fig. 8. The composition of taxpayers in Mexico.

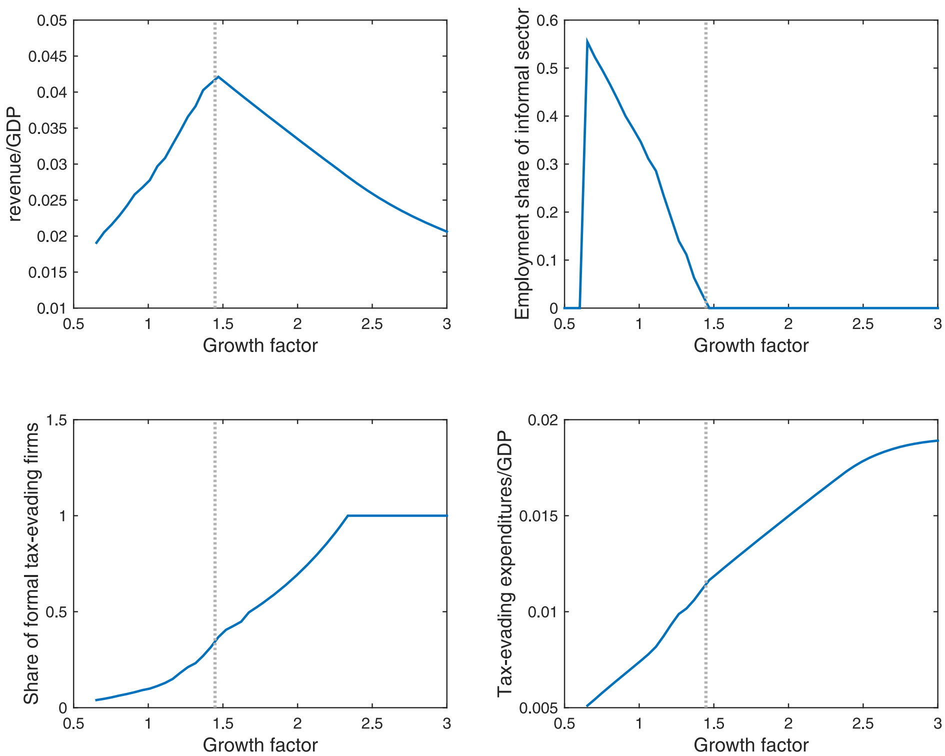

Fig. 9. The effects of development tax revenues, informality, and formal tax evasion.

outward shifts in the support of the distribution of productivities, $F(\cdot)$.[11] This exercise is relevant because recent evidence suggests that informality is a symptom, rather than a cause, of low economic development (LaPorta and Shleifer, 2014).

In Fig. 9, I show the effects of economic development on revenues as a share of GDP, the employment share of the informal sector, the share of formal, tax-evading firms, and tax-evading expenditures as a share of GDP. As $[Z_L, Z_H]$ shifts out, informality becomes less attractive for a growing share of firms. The dotted vertical line indicates a growth factor of 1.44—the minimum growth required to eliminate informality. Beyond that point, tax revenues as a share of GDP steadily decline,

[11] Lucas and Robert (2009), and Perla and Tonetti (2014) provide models where outward shifts in the distribution of individual productivities occur endogenously through the flow of ideas.

the informal sector disappears, while both the share of formal, tax-evading firms and tax-evading expenditures as a share of GDP continue to increase. This happens because the share of formal, tax-evading firms increases as the economy develops. In fact, the minimum growth necessary to eliminate informality also coincides with the highest level of tax collection.

This growth counterfactual exercise also provides a rationale for the existence of developed countries with high tax evasion and/or avoidance. For instance, in 2009, per capita GDP in Greece was 2.07 times that in Mexico. On the other hand, total income tax revenues as a percentage of GDP were 6.9 for Greece, and 5.2 for Mexico, both falling well below the OECD average of 11%.[12]

5. Concluding remarks

I presented a model where informal, formal tax-compliant, and formal tax-evading firms coexist in equilibrium, as a result of the institutional environment. The theory predicts a system of effective tax rates that relies on medium-sized firms to raise revenues. This result links the evidence on the so-called "missing middle" in developing countries to a low capacity to generate income tax revenues. This result is also consistent with recent work by Bi et al. (2016) linking low effective tax rates to low fiscal limits in developing countries, when compared to developed countries.

When the distribution of productivities improves over time, given taxes and enforcement parameters, informality decreases, but formal tax evasion increases, thus limiting the state's capacity to raise tax revenues. If, as suggested by LaPorta and Shleifer (2014), lower informality is a byproduct of development, and not vice versa, then the tax authorities' efforts to raise revenues could be more effective when directed towards formal tax evasion.

In an attempt to keep the theory clear and tractable—but still suitable for quantitative analysis—I have abstracted from some aspects of reality that would be interesting to consider in future work. Specifically, there is evidence that the enforcement of some taxes increases with firm size, as argued by Levy (2008). One could also think about how rent seeking interacts with sources of misallocation other than informality, such as financial constraints, as in López (2014). Last, the limited evidence on firm-level effective income tax rates suggests that the returns to firm's tax-evading efforts are not constant across firms. Allowing for heterogeneity in both productivity and the returns to tax-evasion expenditures would bring us a step closer to understanding the aggregate consequences of having an institutional environment that rewards rent-seeking skills, and of firm-level political and bureaucratic connections.

Appendix A

In this section, I draw from Midrigan and Xu (2014) to analyze the misallocation effects of informality. In doing so, I formalize the numerical findings and discussion in Leal-Ordóñez (2014). Consider the centralized problem of allocating aggregate stocks of capital and labor across firms distributed over the interval $[Z_W, Z_H]$ according to some c.d.f. $\tilde{F}(\cdot)$, so as to maximize total output

$$\max_{\{K(Z),N(Z)\}} Y = \int_{Z_W}^{Z_H} Z^{1-\theta}(K(Z)^{\alpha}N(Z)^{1-\alpha})^{\theta} d\tilde{F}(Z),$$

subject to

$$K = \int_{Z_W}^{Z_H} K(Z) d\tilde{F}(Z),$$

$$N = \int_{Z_W}^{Z_H} N(Z) d\tilde{F}(Z).$$

Let $\tilde{Z} = \int_{Z_W}^{Z_H} Z d\tilde{F}(Z)$. In an economy without informality, capital and labor are allocated so as to equalize returns to each factor across firms, which satisfies, for all Z,

$$K(Z) = \frac{Z}{\tilde{Z}}K,$$

$$N(Z) = \frac{Z}{\tilde{Z}}N.$$

Aggregate output is then equal to

$$Y = TFP^F(K^{\alpha}N^{1-\alpha})^{\theta},$$

where

$$TFP^F = \frac{Y}{(K^{\alpha}N^{1-\alpha})^{\theta}} = \tilde{Z}^{1-\theta}.$$

[12] Data on per capita GDP come from The World Bank (2015a), while data on tax revenues come from the OECD Revenue Statistics.

Now consider an economy where there are some informal firms. The labor and capital allocations of informal firms in this case are

$$K(Z) = \omega_K(Z)\frac{Z}{\tilde{Z}}K,$$
$$N(Z) = \omega_N(Z)\frac{Z}{\tilde{Z}}N.$$

The wedges $\omega_K(Z)$, $\omega_N(Z) \in (0, 1]$ measure the firm-level inefficiency in the allocation of resources due to informality, and are given by

$$\omega_K(Z) = \left(\frac{r}{r+\mu(Z)}\right)^{\frac{1-\theta(1-\alpha)}{1-\theta}},$$
$$\omega_N(Z) = \left(\frac{r}{r+\mu(Z)}\right)^{\frac{\theta\alpha}{1-\theta}}.$$

Aggregate productivity is then

$$TFP^I = \frac{\int_{Z_W}^{Z_I}\omega_Y(Z)Zd\tilde{F} + \int_{Z_I}^{Z_H}Zd\tilde{F}}{\tilde{Z}^{\theta}},$$

where $\omega_Y(Z) = \left[\omega_K(Z)^{\alpha}\omega_N(Z)^{1-\alpha}\right]^{\theta}$. Clearly, as long as there is a set $\mathcal{Z} \subset [Z_W, Z_I]$, such that $\omega_Y(Z) \in (0, 1)$ for $Z \in \mathcal{Z}$, $TFP^I < TFP^F$. In other words, if there are some informal firms constrained by the detection threshold D, there will be aggregate productivity losses due to informality.

Notice that $TFP^I = TFP^F$ both when $D = 0$ and when $D = \infty$. In the first case there is no advantage to informality, so all firms are formal, while in the second, the detection threshold does not create any distortions in the allocation of inputs, so all firms choose informality, without causing any productivity losses. This gives rise to a U-shaped relationship between informality and productivity. Leal-Ordóñez (2014) finds the same result in a numerical exercise. It is not surprising, then, that full enforcement against informality achieves productivity gains. The same can be said about the effects of D on output and the capital stock. Clearly, when $D = 0$ tax revenues are as high as they can be, while when $D = \infty$, they are zero.

Appendix B

Proof of Lemma 1. Suppose an agent has decided to be a formal entrepreneur. He will chose to evade taxes as long as $\Pi_E(Z) > \Pi_C(Z)$, and is indifferent between evading taxes and complying whenever $\Pi_E(Z) = \Pi_C(Z)$. The indifference condition is given by

$$Z_E\Psi(w,r) - \frac{1}{\phi}[1 + \log(\tau_0\phi Z_E\Psi(w,r))] = (1-\tau_0)Z_E\Psi(w,r),$$

which simplifies to

$$\tau_0\Psi(w,r)Z_E = \frac{1+\log(\tau_0\phi\Psi(w,r)Z_E)}{\phi}. \tag{13}$$

Notice that the left hand side of (13) is the cost of being compliant, while the right hand side represents the cost of evading taxes. Let $X = \tau_0\phi\Psi(w,r)Z$. The expression above can then be rewritten as $X = 1 + \log(X)$, which has a unique fixed point at $X^* = 1$. Moreover, this fixed point is also a tangency point, with $X > 1 + \log(X)$ whenever $X > 1$. This condition pins down the productivity cutoff for tax evasion at $Z_E = [\tau_0\phi\Psi(w,r)]^{-1}$. For $Z > Z_E$, the cost of being compliant exceeds the cost of tax evasion. For $Z < Z_E$, $B(Z) < 0$, which violates the non-negativity constraint, while $B(Z_E) = 0$. Therefore, entrepreneurs with ability $Z \le Z_E$ will choose $B(Z) = 0$, and pay the statutory tax rate, while those with ability $Z > Z_E$ will pay $B(Z) > 0$ in order to evade taxes. This concludes the proof. □

Proof of Lemma 2. Assume there is a $Z_P > 0$ such that $\omega(Z_P) > (1-\tau_0)$.

Case I: $Z_I < Z_E$. I start with the case in which there are some formal firms that chose to comply. In this case, the indifference condition is given by $\Pi_I(Z_I) = \Pi_C(Z_I)$, or

$$\omega(Z_I)Z_I\Psi(w,r) = (1-\tau_0)Z_I\Psi(w,r),$$

which simplifies to

$$\omega(Z_I) = (1-\tau_0)$$

The function $\omega(Z) \in [0, 1]$ is decreasing, and strictly decreasing in (0, 1).[13] Since there is a $Z_P > 0$ such that $\omega(Z_P) > (1-\tau_0)$, by the Intermediate Value Theorem, there exists a unique Z_I such that $\omega(Z_I) = (1-\tau_0)$.

[13] As Z increases, the size constrain binds and $\mu(Z)$ increases, so $\omega(Z)$ decreases.

Case II: $Z_I > Z_E$. In this case only informal and tax-evading firms exist. First notice that we can write the profits of informal firms that are bound by the detection threshold as

$$\Pi_I(Z) = (1-(1-\alpha)\theta)\left[\frac{(1-\alpha)\theta}{w}\right]^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}} D^{\frac{\alpha\theta}{1-(1-\alpha)\theta}} Z^{\frac{1-\theta}{1-(1-\alpha)\theta}} - rD$$

Since there are no formal tax-complaint firms, it has to be that $\Pi_I(Z_E) > \Pi_E(Z_E)$. As Z increases and the detection threshold binds, the profit function $\Pi_I(Z)$ will be strictly increasing and strictly concave. Since $\Pi_E(Z)$ is strictly increasing and stricly convex for $Z > Z_E$, it crosses $\Pi_I(Z)$ only once, at Z_I. This concludes the proof. □

Proof of Lemma 3. The profit function $\Pi^*(Z;\, w,\, \phi,\, \tau_0,\, D)$ is strictly increasing in Z with $\Pi^*(0; w, \phi, \tau_0, D) = 0$, so for any $w > 0$ there is a unique $Z_W > 0$ such that $\Pi^*(Z_W; w, \phi, \tau_0, D) = w$. Clearly, $V(Z) = \Pi^*(Z; w, \phi, \tau_0, D)$ for any $Z \geq Z_W$ and $V(Z) = w$ otherwise. □

Appendix C

The main body of this article focuses on the effects of a tax on profits, which are well-known, from a theoretical standpoint, to be non-distortionary (at least in a partial equilibrium setting). In this section, I consider a tax alternative: an all-grabbing tax on output, similar to those studied in the literature on misallocation and TFP, as in Restuccia and Rogerson (2008), Hsieh and Klenow (2009), and Leal-Ordóñez (2014). A tax on output will affect the input choices by formal firms, whether they engage in tax evasion or not. Moreover, the choices of inputs and expenditures on tax evasion are not independent from each other.[14]

The problem faced by a formal firm is to choose (K_F, N_F, B), given Z, τ_0, ϕ and factor prices (w, r), to maximize profits,

$$\Pi_F(Z) \equiv \max_{K_F, N_F, B} \left(1-\tau_0 e^{-\phi B}\right) Z^{1-\theta}\left(K_F^{\alpha} N_F^{1-\alpha}\right)^{\theta} - rK_F - wN_F - B,$$

subject to $B \geq 0$.

The optimal choices by the entrepreneur are

$$K_F(Z) = Z\left(1-\tau_0 e^{-\phi B}\right)\left[\frac{\alpha\theta}{r}\kappa^{-\theta(1-\alpha)}\right]^{\frac{1}{1-\theta}}, \tag{14}$$

$$N_F(Z) = Z\left(1-\tau_0 e^{-\phi B}\right)\left[\frac{(1-\alpha)\theta}{w}\kappa^{\theta\alpha}\right]^{\frac{1}{1-\theta}}, \tag{15}$$

$$B(Z) = \frac{1}{\phi}\ln\left(\tau_0 \phi Z^{1-\theta}\left(K_F^{\alpha} N_F^{1-\alpha}\right)\right), \tag{16}$$

Where

$$\kappa \equiv \frac{K_F}{N_F} = \left(\frac{\alpha}{1-\alpha}\right)\frac{w}{r}$$

The solution to the input and tax evasion expenditures choices requires solving the above system of non-linear equations numerically. Even though some analytical convenience is lost under a tax on output, the productivity thresholds derived above still exist. Fig. 10 shows the firm type and occupational choices under a tax on output.

I recalibrate the model following the previous strategy. The only change is the target moment for tax collection. Instead of matching corporate tax revenues as a share of GDP, the model is calibrated to match all tax revenues originated from firms, as a share of GDP. These include payroll, sales, and profit taxes, plus some special taxes on production. According to Mexican tax authorities, all taxes collected from firms amount to 7% of GDP. Table 7 contains the calibrated parameter values, while Table 8 shows the calibration results. The model performs fairly well at matching both targeted and some non-targeted moments.

In order to explore the aggregate effects of tax evasion under a tax on output, I solve the model for different values for ϕ and τ, while keeping the rest of the parameters as in the baseline calibration. Table 9 shows the aggregate effects for different values of ϕ, starting from of a low value (half of that in the baseline calibration), to a moderately large one (1.5 times the value in the baseline calibration), and then for more extreme values.

The results highlight the trade-off between a stricter tax system and economic performance. When the returns parameter, ϕ, is lower than its baseline value, TFP, aggregate capital, output and the equilibrium wage rate are all below their baseline counterparts. Tax revenues are higher when ϕ is lower, even though some firms are pushed into the informal sector. As the returns to tax evasion increase, aggregate productivity, the capital stock, and output increase. As firms expand, they demand

[14] Thus, when we consider a tax on output, tax evasion has distortionary effects in both partial and general equilibrium settings.

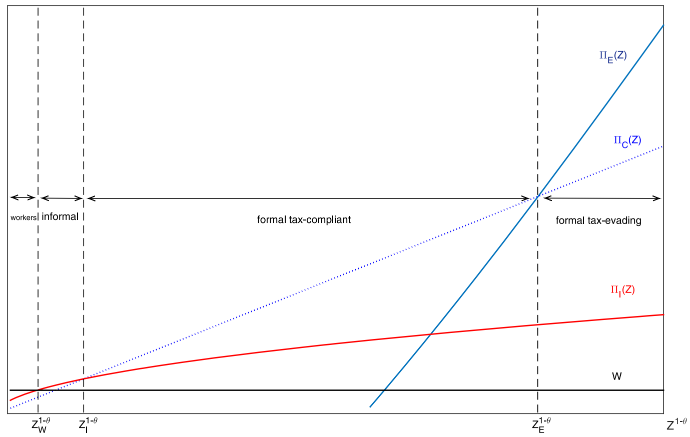

Fig. 10. Firm type and occupational choices with a tax on output.

Table 7
Parameter values for output tax model.

Parameter	Value
Assigned	
α	0.3
δ	0.07
τ_0	0.3
θ	0.76
$Z_L^{1-\theta}$	1
Calibrated (jointly)	
D	2.175
ϕ	0.255
$Z_H^{1-\theta}$	9.54
s	7.144
β	0.98

Table 8
Calibration results for output tax model.

Targeted	Data	Model
Size of informal sector (% of emp.)	0.34	0.35
Revenues (% of GDP)	0.07	0.07
$\left(\frac{N_{max}}{N_{min}}\right)^{1-\theta}$	9.5	9.44
Average firm size	5.5	5.6
Capital-output ratio	1.9	1.9
Non-targeted	Data	Model
Share of firms of size $<= 10$	0.94	0.96
Share of firms of size between 11 and 50	0.05	0.025
Share of firms of size between 51 and 250	0.008	0.013
Share of firms of size > 250	0.002	0.002

Table 9
Returns to tax evasion with a tax on output.

Aggregate	$\phi = 0.1276$	$\phi = 0.3828$	$\phi = 1$	$\phi = 5$
TFP*	0.940	1.066	1.369	1.631
K-stock*	0.817	1.183	2.067	3.290
Output*	0.889	1.117	1.670	2.232
Wage*	0.980	1.012	1.064	1.207
Informal sector (% of employment)	0.410	0.315	0.194	0.000
Tax revenues (% of GDP)	0.087	0.056	0.017	0.003
Share of tax-compliant firms	0.220	0.212	0.107	0.000
Share of informal firms	0.775	0.759	0.713	0.000
Share of tax-evading firms	0.004	0.029	0.181	1.000
Rent-seeking (% of GDP)	0.019	0.035	0.056	0.053

*Values relative to baseline. $\phi = 0.255$ in baseline calibration.

Table 10
Alternative statutory output tax rates.

Aggregate	$\tau_0 = 0.15$	$\tau_0 = 0.2$	$\tau_0 = 0.25$	$\tau_0 = 0.35$	$\tau_0 = 0.4$
TFP*	1.066	1.127	1.102	0.973	0.957
K-stock*	1.523	1.584	1.382	0.867	0.804
Output*	1.173	1.262	1.196	0.942	0.910
Wage*	1.132	1.056	1.010	0.993	0.994
Informal sector (% of employment)	0.000	0.000	0.124	0.467	0.522
Tax revenues (% of GDP)	0.053	0.077	0.089	0.041	0.010
Share of tax-compliant firms	0.997	0.993	0.656	0.065	0.006
Share of informal firms	0.000	0.000	0.333	0.918	0.975
Share of tax-evading firms	0.003	0.007	0.012	0.017	0.019
Rent-seeking (% of GDP)	0.006	0.013	0.021	0.036	0.040

*Values relative to baseline. $\tau_0 = 0.3$ in baseline calibration.

more labor, which pushes the equilibrium wage upwards. Previously informal firms find it more profitable to join the formal sector, so informality decreases as ϕ increases. Tax revenues, however, decrease, because a growing share of firms make the leap from tax compliance to formal tax evasion, including those who left the informal sector. For very high values of ϕ, all firms in the economy engage in tax evasion.

Table 10 shows the results from solving the model for different values of the statutory tax rate, τ_0, keeping the rest of the parameters as in the baseline calibration. Aggregate productivity, the capital stock, output and the equilibrium wage are all higher for values of τ_0 below its calibrated value of 0.3. Lower taxes encourage informal firms to join the formal sector, and discourage formal firms from engaging in tax evasion, although not entirely. This results in a wider tax base, but not necessarily higher tax revenues. Thus, tax revenues exhibit a Laffer-curve property with respect to the statutory tax rate: they increase with the tax rate up to a point, after which they start to decrease. Notice that tax revenues are higher for $\tau_0 = 0.25$ than with the calibrated value of $\tau_0 = 0.3$. Furthermore, when $\tau_0 = 0.25$, the economy performs better than under the baseline tax rate: TFP, capital, output and wages are higher, and the informal sector is smaller. Last, as the tax rate increases, the amount of resources devoted to tax evasion increase, which is consistent with the theoretical results presented above.

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